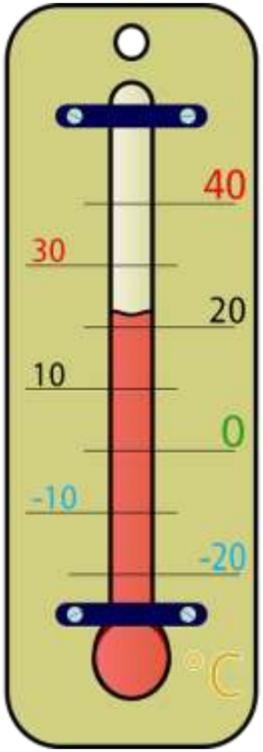
Advanced Mathematics
Support Programme®

Did you know?

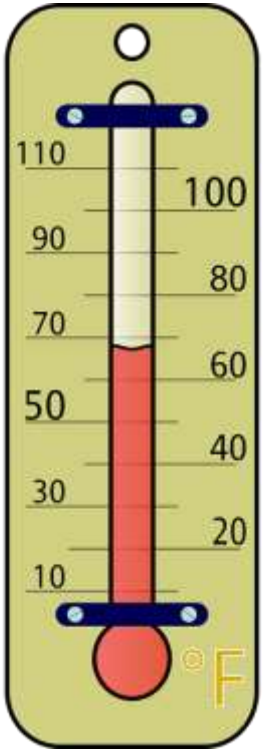
This is a well known formula that you might recognise.



$$F = \frac{9}{5}C + 32$$

It is used to change temperatures in degrees Celsius ^{o}C to degrees Fahrenheit ^{o}F

For example: If it is 20^{o}C to find the temperature in ^{o}F you simply substitute C=20 into the formula above: 68^{o}F



What would I need to do if I wanted to convert from Fahrenheit to Celsius??

Rearranging 1

1. Solve $3x + 25 = 60$

2. Rearrange $z = w + 3$ to make w the subject

3. Rearrange $5x - 4 = 2y$ to make x the subject

4. Rearrange $y = \frac{t}{6}$ to make t the subject

5. $y = 6p^2 + 2$ rearrange to make p the subject

6. The area of a circle is found using $\text{A} = \pi r^2$ Write the equation you would use to find the radius.

7. In a right angled triangle $sinx = \frac{Opp}{Hyp}$ write down the equation for finding the opposite side.

8. To change temperatures in Celsius to Fahrenheit this formula is used.

$$F = \frac{9}{5}C + 32$$

Rearrange to give the formula for converting Celsius to Fahrenheit

Rearranging 1

Solutions on the next slide….

Rearranging 1 Solutions

1. Solve $3x + 25 = 60$ → $3x = 60 - 25$
   $3x = 35$
   $x = \frac{35}{3}$

2. Rearrange $z = w + 3$ to make w the subject → $z - 3 = w$
   $or\ w = z - 3$

3. Rearrange $5x - 4 = 2y$ to make x the subject → $5x = 2y + 4$
   $x = \frac{2y + 4}{5}$

4. Rearrange $y = \frac{t}{6}$ to make t the subject → $6y = t$
   $or\ t = 6y$

Unsure about any of these? Search Rearranging Formulae. Next try Skills check 2….

Rearranging 1 Solutions

5. $y = 6p^2 + 2$ rearrange to make p the subject

$$y - 2 = 6p^2 \qquad p^2 = \frac{y-2}{6}$$

$$p = \pm\sqrt{\frac{y-2}{6}}$$

6. The area of a circle is found using $A = \pi r^2$ Write the equation you would use to find the radius.

$$\frac{A}{\pi} = r^2 \qquad r = \sqrt{\frac{A}{\pi}}$$

7. In a right angled triangle $sinx = \frac{Opp}{Hyp}$ write down the equation for finding the opposite side.

$$Opp = Hyp \times sinx$$

8. To change temperatures in Celsius to Fahrenheit this formula is used.

$$F = \frac{9}{5}C + 32$$

Rearrange to give the formula for converting Celsius to Fahrenheit

$$F = \frac{9}{5}C + 32$$

$$F - 32 = \frac{9}{5}C$$

$$5(F - 32) = 9C$$

$$\frac{5}{9}(F - 32) = C$$

Unsure about any of these? Search Rearranging Formulae. Next try Skills check 2….

Rearranging 2

1. Make x the subject of $x - f = y + b$

2. Make y the subject $ty - x^2 = b$

3. Make c the subject $ac + d = m^2$

4. Make a the subject $x(a - e) = d$

5. Make y the subject $b(y - b) = b^2$

6. To find velocity, v, we use the formula

$$v^2 = u^2 - 2as$$

   Rearrange to find s

7. The area of a sector of a circle is given by $A = \frac{\theta \pi r^2}{360}$ Express θ in terms of A, π and r

8. Make x the subject $m(y - x) = t$

Rearranging 2

Solutions on the next slide….

Rearranging 2 Solutions

1. Make x the subject of $x - f = y + b$ → $x = y + b + f$

2. Make y the subject $ty - x^2 = b$ →

$$ty = b + x^2$$
$$y = \frac{b + x^2}{t}$$

3. Make c the subject $ac + d = m^2$ →

$$ac = m^2 - d$$
$$c = \frac{m^2 - d}{a}$$

4. Make a the subject $x(a - e) = d$ →

$$xa - xe = d$$
$$xa = d + xe$$
$$a = \frac{d + xe}{x}$$

or

$$a - e = \frac{d}{x}$$
$$a = \frac{d}{x} + e$$

Can you see that these are equivalent?

Rearranging 2 Solutions

5. Make y the subject $b(y - b) = b^2$

$$by - b^2 = b^2$$
$$by = 2b^2$$
$$y = 2b$$

Solution

6. To find velocity, v, we use the formula
$$v^2 = u^2 - 2as$$
Rearrange to find s

$$v^2 + 2as = u^2$$
$$2as = u^2 - v^2$$
$$s = \frac{u^2 - v^2}{2a}$$

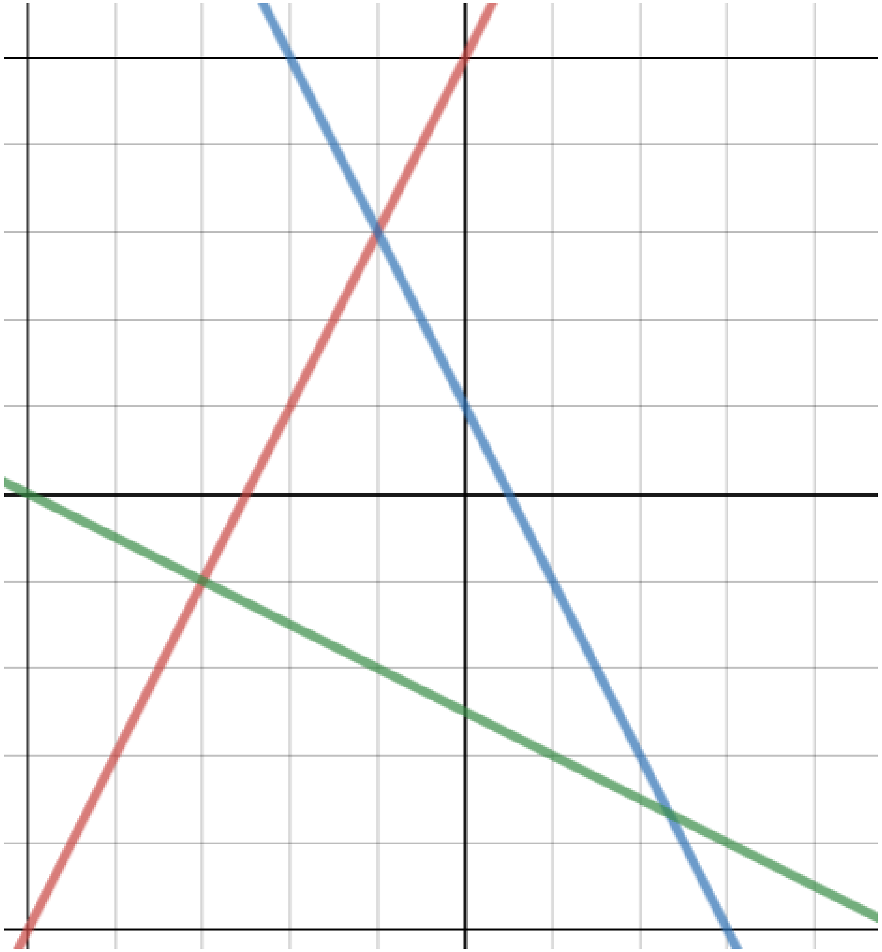
7. The area of a sector of a circle is given by $A = \frac{\theta\pi r^2}{360}$ Express θ in terms of A, π and r

$$360A = \theta\pi r^2$$
$$\theta\pi r^2 = 360A$$
$$\theta = \frac{360A}{\pi r^2}$$

8. Make x the subject $m(y - x) = t$

$$my - mx = t$$
$$my = t + mx$$
$$mx = my - t$$
$$x = \frac{my - t}{m}$$

Line them up 1



Which is which?

- $y = 2x + 5$
- $2y + x + 5 = 0$
- $y + 2x = 1$

How does rearranging enable you to justify your answer?

Line them up 1 Solution

Which is which?

- $y = 2x + 5$
- $2y + x + 5 = 0$
- $y + 2x = 1$

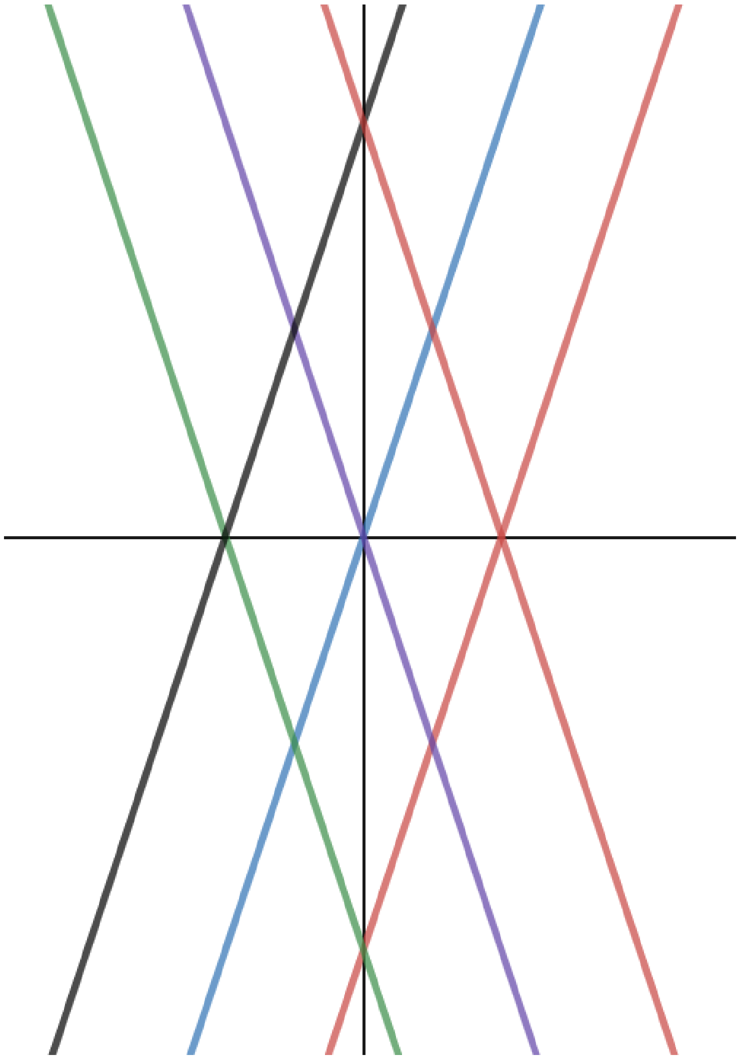
Why?

- $y = 2x + 5$
- $y = -\frac{x}{2} - \frac{5}{2}$
- $y = -2x + 1$

By rearranging into the form $\boldsymbol{y = mx + c}$ you can easily compare the gradient and intercept of each line.

Line them up 2

Label the lines with these equations.



$$y = 4 - 3x$$

$$y + 3x + 4 = 0$$

$$y + 3x = 0$$

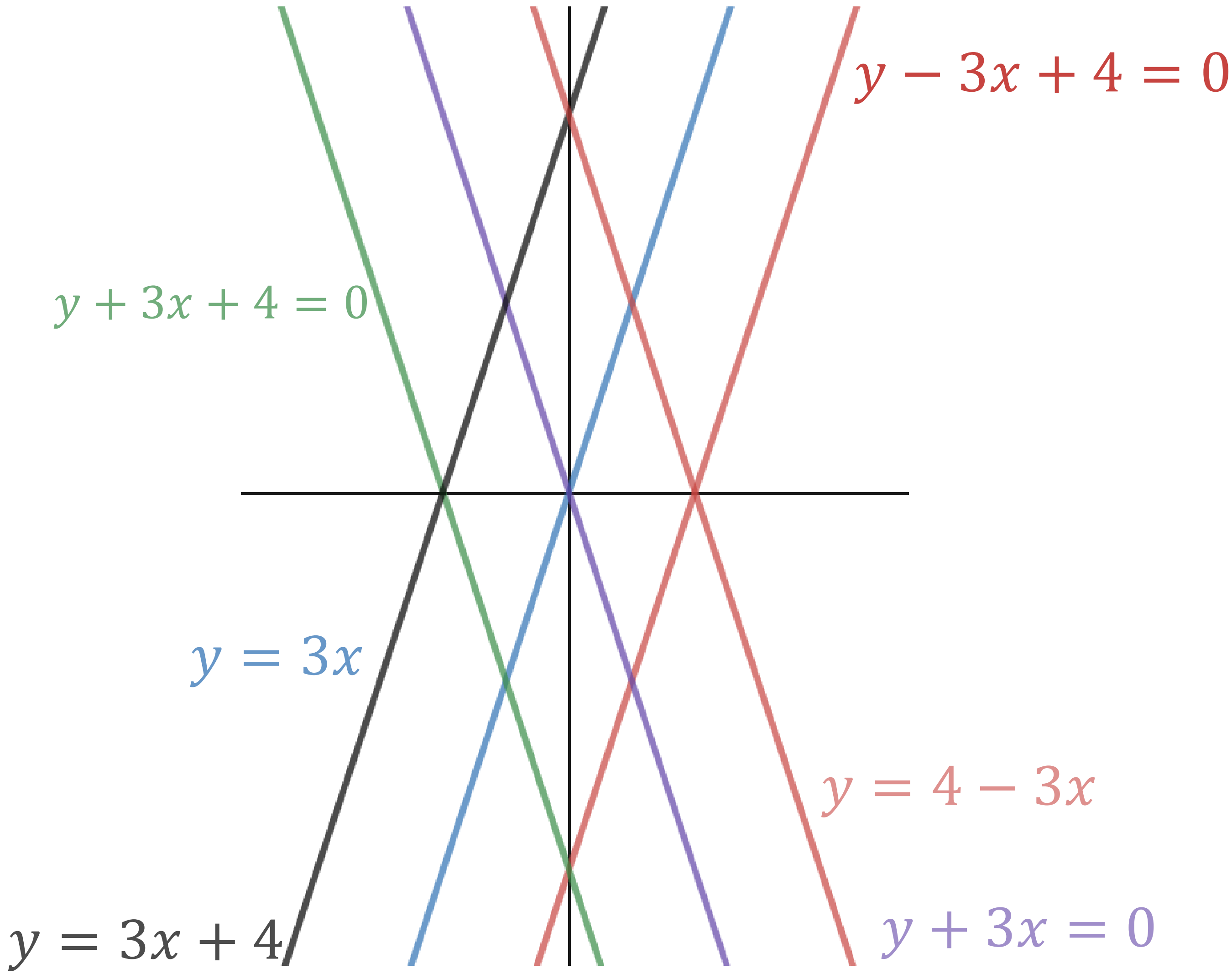
$$y = 3x$$

$$y = 3x + 4$$

$$y - 3x + 4 = 0$$

Line them up 2 Solution

Label the lines with these equations.



Pairing up

Can you sort the cards into pairs under the following headings:

- These lines are perpendicular
- These lines have the same x intercept
- These lines have the same y intercept
- These lines are parallel
- These lines go through the point (1,5)
- These lines…

$3y = 2x - 8$	$y = -(x + 8)$	$y = 4x + 4$
$2y + x = 4$	$y = 6x - 4$	$y = 8x - 3$
$y + x + 8 = 0$	$2y = 8x + 3$	$4y = x + 3$
$2y + 8 = 3x$	$y + 6x = 11$	$y + 4x + 6 = 0$

Pairing up Solution

Can you sort the cards into pairs under the following headings:

- These lines are perpendicular

$4y = x + 3$	$y + 4x + 6 = 0$

- These lines are parallel

$y = 4x + 4$	$2y = 8x + 3$

- These lines have the same y intercept

$2y + 8 = 3x$	$y = 6x - 4$

- These lines have the same x intercept

$2y + x = 4$	$3y = 2x - 8$

- These lines go through the point (1,5)

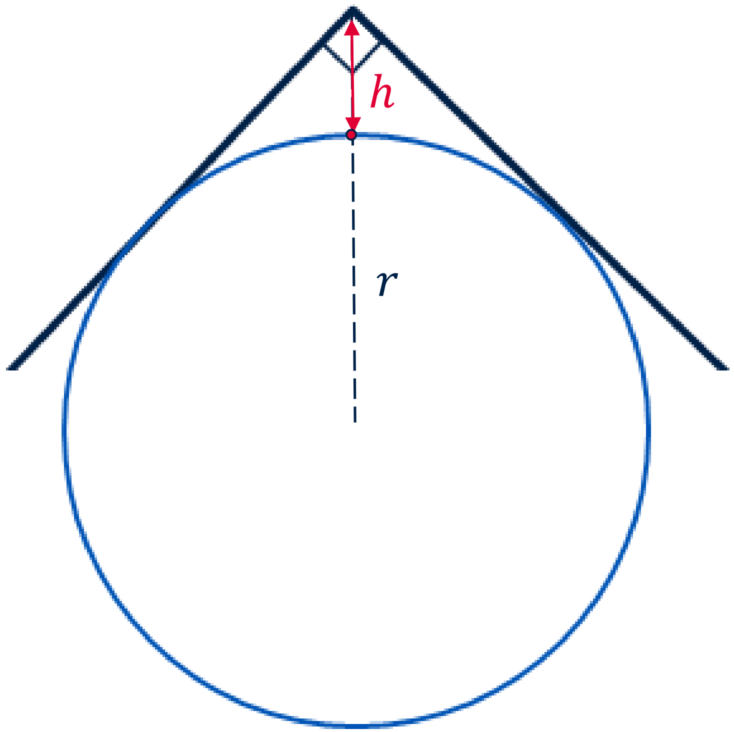
$y = 8x - 3$	$y + 6x = 11$

- These lines are the same line

$y + x + 8 = 0$	$y = -(x + 8)$

Pipe Problem

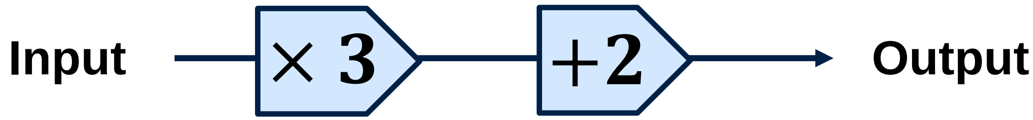
Can you find the radius of the pipe shown if the only measurement you can take is the one marked h?



Click here to watch a video with hints and the solution

A function relates an input to an output

Here is an example of a function machine



Complete the following table for the function machine shown

Input	Output
5	
-4	
x	
	17
	x

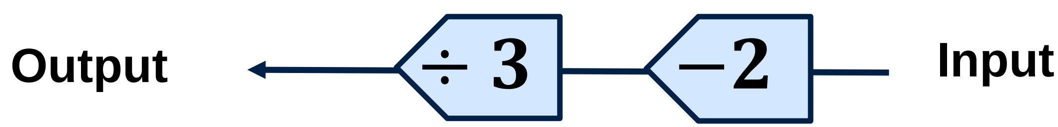
What do you notice?

A function relates an input to an output

Input	Output
5	17
-4	-10
x	$3x + 2$
5	17
$\frac{x-2}{3}$	x

An inverse function goes the other way

To reverse the process inverse operations are used.



Important! The inverse should give us back the original value

Rearranging and Functions Solutions

Let's introduce function notation that you will use in A level maths:

$f(5) = 3 \times 5 + 2 = 17$

Input	Output
5	17
-4	-10
x	$3x + 2$
← $\div 3$ ← -2 ←	
5	17
$\frac{x-2}{3}$	x

$f(5) = 3 \times 5 + 2 = 17$

$f(-4) = 3 \times -4 + 2 = -10$

$f(x) = 3x + 2$

An inverse function goes the other way

$f^{-1}(17) = (17 - 2)/3 = 5$

$f^{-1}(x) = \frac{x-2}{3}$

Important! The inverse should give us back the original value

Lets check: $f(5) = 17 \text{ and } f^{-1}(17) = 5$

Rearranging and Functions

Original function

$$f(x) = 3x + 2$$

Inverse function

$$f^{-1}(x) = \frac{x-2}{3}$$

Find the inverse of each of these functions.

1. $f(x) = 3x - 5$
2. $f(x) = 4x + 7$
3. $f(x) = \frac{x}{2} + 1$
4. $f(x) = \frac{x+2}{3}$
5. $f(x) = \frac{2}{3}x + 3$
6. $f(x) = 3 - 2x$
7. $f(x) = x^2$
8. $f(x) = \sqrt{x+1}$

Instead of reversing a function machine - try re-arranging the original function to make x the subject

Rearranging and Functions

Solutions on the next slide….

Rearranging and Functions

Find the inverse of each of these functions.

1. $f(x) = 3x - 5$ $\qquad f^{-1}(x) = \dfrac{x+5}{3}$

2. $f(x) = 4x + 7$ $\qquad f^{-1}(x) = \dfrac{x-7}{4}$

3. $f(x) = \dfrac{x}{2} + 1$ $\qquad f^{-1}(x) = 2(x-1)$

4. $f(x) = \dfrac{x+2}{3}$ $\qquad f^{-1}(x) = 3x - 2$

*Be careful not to get the notation f^{-1} mixed up with reciprocals and negative powers!

Rearranging and Functions

Find the inverse of each of these functions.

5. $f(x) = \frac{2}{3}x + 3$ $\quad f^{-1}(x) = \frac{3(x-3)}{2}$

6. $f(x) = 3 - 2x$ $\quad f^{-1}(x) = \frac{3-x}{2}$

7. $f(x) = x^2$ $\quad f^{-1}(x) = \pm\sqrt{x}$

8. $f(x) = \sqrt{x+1}$ $\quad f^{-1}(x) = x^2 - 1$

If you want to explore functions further then click here.

Still want more?

Read – Ten key reasons why developing algebraic skills is so important!

Discover more about the graphs of a function and its inverse by exploring this GeoGebra activity.

Watch and learn how maths, in particular the correct use of brackets, influences music, poetry and even rap!

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