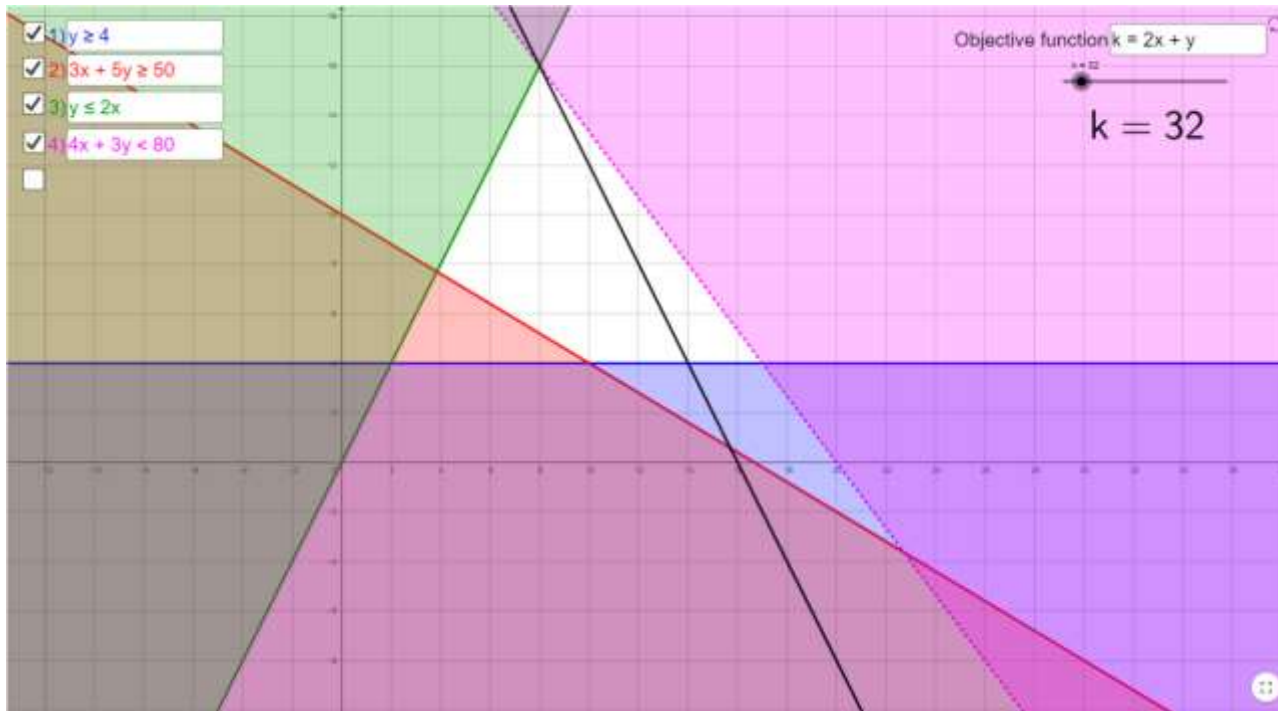




**Advanced Mathematics
Support Programme®**

Linear programming is a method that involves solving a set of linear equations or inequalities in order to find the best solution.



It is very useful in industry for finding the best level of production, or the maximum profit depending on varying costs, sales, mix of products or availability of labour etc...



Solve the following:

1. $8x - 3 = 5x + 13$

2. $3x + 1 > 10$ and $2x + 7 < 15$

3. $3(x + 6) > 12$

4. $24 - 3x = 9$

5. $14 \geq 8 + 5x$

6. $6 - 2x < 5x + 34$

7. $\frac{2x + 3}{7} = \frac{4x - 5}{3}$

8. The perimeter of the rectangle is 24cm. Find the value of x

x cm



$2x + 2$ cm



Solving Linear 1



Solutions on the next slide....



1. $8x - 3 = 5x + 13$



$$\begin{aligned} 3x - 3 &= 13 \\ 3x &= 16 \\ x &= \frac{16}{3} \end{aligned}$$

2. $3x + 1 > 10$
and $2x + 7 < 15$



$$\begin{aligned} 3x &> 9 & 2x &< 8 \\ x &> 3 & x &< 4 \end{aligned}$$

So $3 < x < 4$

3. $3(x + 6) > 12$



$$\begin{aligned} x + 6 &> 4 & \text{or} & 3x + 18 > 12 \\ x &> -2 & & 3x > -6 \\ & & & x > -2 \end{aligned}$$

4. $24 - 3x = 9$



$$\begin{aligned} -3x &= -15 \\ x &= 5 \end{aligned}$$



5. $14 \geq 8 + 5x$



$$6 \geq 5x$$

$$\frac{6}{5} \geq x \text{ or } x \leq \frac{6}{5}$$

6. $6 - 2x < 5x + 34$



$$6 < 7x + 34$$

$$-28 < 7x$$

$$-4 < x \text{ or } x > -4$$

7. $\frac{2x + 3}{7} = \frac{4x - 5}{3}$



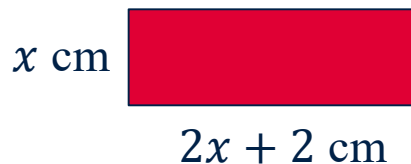
$$3(2x + 3) = 7(4x - 5)$$

$$6x + 9 = 28x - 35$$

$$44 = 22x$$

$$x = 2$$

8. The perimeter of the rectangle is 24cm. Find the value of x



$$x + (2x + 2) + x + (2x + 2) = 24$$

$$6x + 4 = 24$$

$$6x = 20$$

$$x = \frac{10}{3}$$



Solve the following:

1. $6x + 5 = 47$

2. $5x + 7 = x + 25$

3. $7(x - 4) = 14$

4. $29 - 4x < 22$

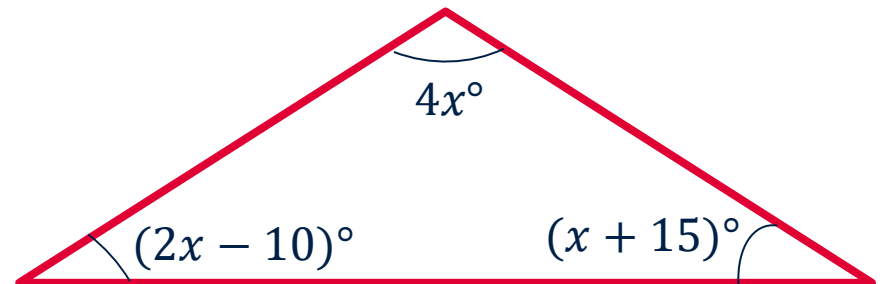
5. $3x < 2x - 1 < 4x + 2$

Hint: Split into two inequalities

6. $19 + 2x = 3x + 15$

7. $\frac{3x - 1}{5} \geq \frac{3x + 5}{2}$

8. Find the value of x





Solving Linear 2



Solutions on the next slide....



Solve the following:

1. $6x + 5 = 47$



$$6x = 42$$

$$x = 7$$

2. $5x + 7 = x + 25$



$$4x + 7 = 25$$

$$4x = 22$$

$$x = 5.5$$

3. $7(x - 4) = 14$



$$x - 4 = 2 \quad \text{or} \quad 7x - 28 = 14$$

$$x = 6 \quad \quad \quad 7x = 42$$

$$\quad \quad \quad \quad \quad \quad \quad x = 6$$

4. $29 - 4x < 22$



$$29 - 22 < 4x \quad \text{or} \quad -4x < -7$$

$$7 < 4x \quad \quad \quad x > \frac{-7}{-4}$$

$$\frac{7}{4} < x \quad \quad \quad x > \frac{7}{4}$$

When multiplying or dividing by a negative, the inequality reverses direction



5. $3x < 2x - 1 < 4x + 2$



$$3x < 2x - 1$$

$$x < -1$$

$$2x - 1 < 4x + 2$$

$$-1 < 2x + 2$$

$$-3 < 2x$$

$$-\frac{3}{2} < x$$

So $-\frac{3}{2} < x < -1$

6. $19 + 2x = 3x + 15$



$$19 = x + 15$$

$$4 = x$$

7. $\frac{3x - 1}{5} \geq \frac{3x + 5}{2}$



$$2(3x - 1) \geq 5(3x + 5)$$

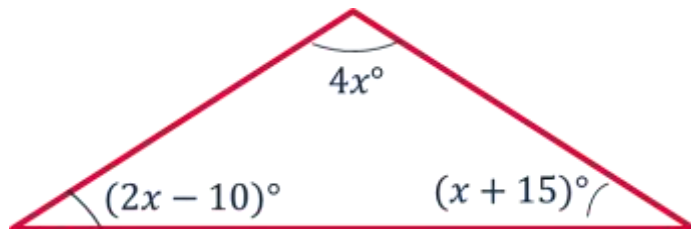
$$6x - 2 \geq 15x + 25$$

$$-2 \geq 9x + 25$$

$$-27 \geq 9x$$

$$-3 \geq x$$

8. Find the value of x



$$4x + (2x - 10) + (x + 15) = 180$$

$$7x + 5 = 180$$

$$7x = 175$$

$$x = 25$$



The number in the middle of each group of 3 adjoining cells is the average of its two neighbours.

5			23	
----------	--	--	-----------	--

What number goes in the right hand cell?



The number in the middle of each group of 3 adjoining cells is the average of its two neighbours.

What number goes in the right hand cell?

5			23	
---	--	--	----	--

As with most problems it is a good idea to begin by trying some numbers in the problem and see if that gives you a greater understanding.

In order for 23 to be the average of the numbers in the cells either side they must both be the same distance from 23 e.g.

16	23	30
$23-7$		$23+7$

Extending that to the other cells would mean:

BUT 9 is not the average of 5 and 16!

5	9	16	23	30
	$16-7$		$16+7$	

Let's generalise to try and form an equation:

$23-3x$	$23-2x$	$23-x$	23	$23+x$
	$+x$	$+x$	$+x$	$+x$

There are a few different equations that we could form...

Here is one example. $23-3x = 5$

We can solve to get:
 $18 = 3x$
 $x = 6$

5	11	17	23	29
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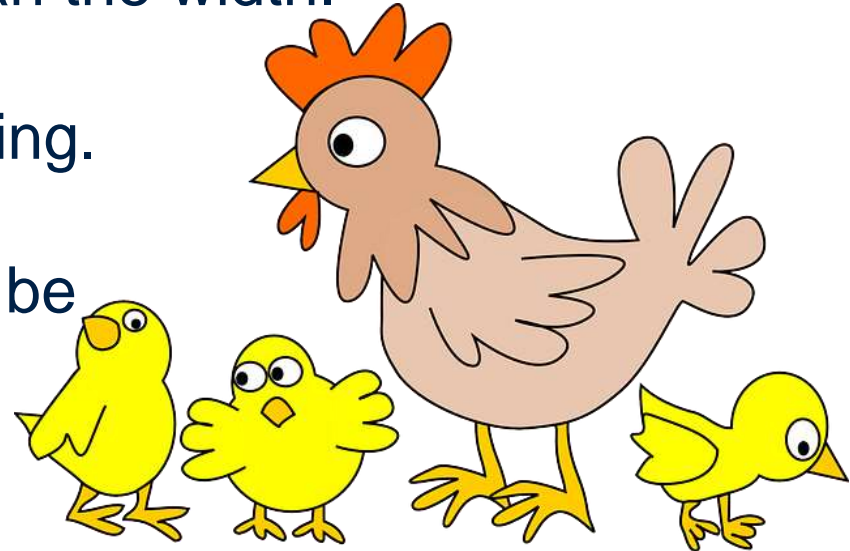


Victoria has just bought some chickens. She wants to keep them safe in a small enclosure.

The enclosure will be a rectangle where the length is 3m longer than the width.

Victoria has only got 30m of fencing.

The area of the enclosure has to be greater than 20m^2

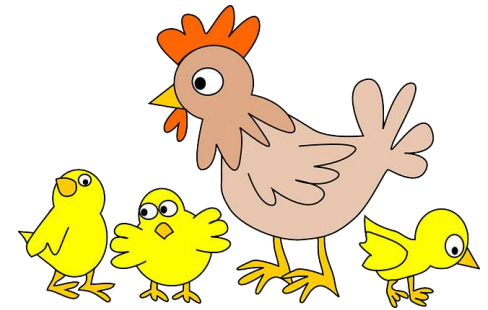


The length and width are integers.

How many different size enclosures can Victoria make?

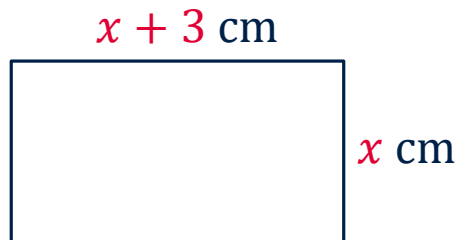


- The enclosure will be a rectangle
- The length is 3m longer than the width.
- Victoria has only got 30m of fencing.
- The area of the enclosure has to be greater than 20m^2
- The length and width are integers.



How many different size enclosures can Victoria make?

Draw a diagram



Write an inequality for the maximum perimeter of the fence

$$2(x + 3) + 2x \leq 30$$

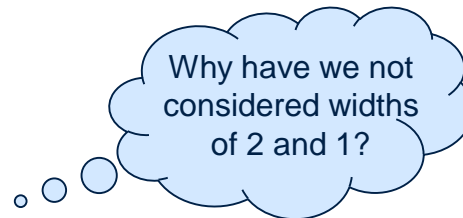
$$4x + 6 \leq 30$$

$$4x \leq 24$$

$$x \leq 6 \quad \leftarrow \text{The maximum width of the enclosure}$$

Consider the area of the enclosure for different possible lengths and widths

Width (m)	Length (m)	Area (m^2)
6	9	54
5	8	40
4	7	28
3	6	18



So Victoria can make 3 different sized enclosures with an area greater than 20m^2



Can you decode this message?

12	1	4	7	5	3	12	4	2	5				
7	4	3	3	6	15	4	9	2	6	9	8	4	10

Solve the equations in the boxes below. Each letter will have a different positive integer solution between 0 and 16.

$$\frac{4r}{d-4} + \frac{2h}{s} = 2$$

$$\frac{g-9}{y+4} = \frac{2}{3}$$

$$3rh + m = 13$$

$$\frac{4g}{5} = 12$$

$$\frac{2c-5+3(c-2)}{2c-1} = 2$$

$$e^3 < 72$$

$$\frac{s+3y}{8s} = \frac{3}{4}$$

$$\frac{6k}{s} - 5 = 11$$

$$100 < t^2 < 169$$

$$\frac{8}{3a} = \frac{4}{a+3}$$

$$\frac{6r+8}{y} = 4$$

$$2(3m+4) = 7m+1$$



Can you decode this message?

$$\begin{array}{cccccc} 12 & 1 & 4 & & 7 & 5 & 3 & 12 & 4 & 2 & 5 \\ 7 & 4 & 3 & 3 & 6 & 15 & 4 & & 9 & 2 & 6 & 9 & 8 & 4 & 10 \end{array}$$

Solve the equations in the boxes below. Each letter will have a different positive integer solution between 0 and 16.

Solve the equations in the following order:

$$1. \quad \frac{4g}{5} = 12$$

$$2. \quad \frac{2c - 5 + 3(c - 2)}{2c - 1} = 2$$

$$3. \quad \frac{8}{3a} = \frac{4}{a + 3}$$

$$4. \quad \frac{g - 9}{y + 4} = \frac{2}{3}$$

$$5. \quad \frac{6r + 8}{y} = 4$$

$$6. \quad 2(3m + 4) = 7m + 1$$

$$7. \quad \frac{s + 3y}{8s} = \frac{3}{4}$$

$$8. \quad \frac{6k}{s} - 5 = 11$$

$$9. \quad 3rh + m = 13$$

$$10. \quad \frac{4r}{d - 4} + \frac{2h}{s} = 2$$

$$11. \quad 100 < t^2 < 169$$

$$12. \quad e^3 < 72$$



Can you decode this message?

12 1 4 7 5 3 12 4 2 5
7 4 3 3 6 15 4 9 2 6 9 8 4 10

Solve the equations in the boxes below. Each letter will have a different positive integer solution between 0 and 16.

$$\frac{4g}{5} = 12 \quad g = 15$$

$$\frac{2c - 5 + 3(c - 2)}{2c - 1} = 2 \quad c = 9$$

$$\frac{8}{3a} = \frac{4}{a + 3} \quad a = 6$$

$$\frac{g - 9}{y + 4} = \frac{2}{3} \quad y = 5$$

$$\frac{6r + 8}{y} = 4 \quad r = 2$$

$$2(3m + 4) = 7m + 1 \quad m = 7$$

$$\frac{s + 3y}{8s} = \frac{3}{4} \quad s = 3$$

$$\frac{6k}{s} - 5 = 11 \quad k = 8$$

$$3rh + m = 13 \quad h = 1$$

$$\frac{4r}{d - 4} + \frac{2h}{s} = 2 \quad d = 10$$

$$100 < t^2 < 169 \quad 10 < t < 13$$

$$e^3 < 72 \quad e < 4.16..$$

“The mystery message cracked”



There are two main ways to solve simultaneous equations.

Elimination

$$3x + 2y = 9$$

$$5x - 2y = -1$$

Add the two equations together to **eliminate** y

$$8x = 8$$

$$x = 1$$

Now we have a value for x we can put it into one of the original equations to find y

$$3 \times 1 + 2y = 9$$

$$3 + 2y = 9$$

$$2y = 6$$

$$y = 3$$

Substitution

$$y + 3x = 5$$

$$2y + 7x = 11$$

Rearrange the first equation in terms of y and then **substitute** into the second equation

$$2(5 - 3x) + 7x = 11$$

$$10 - 6x + 7x = 11$$

$$x = 1$$

Now we have a value for x we can put it into one of the original equations to find y

$$y + 3 \times 1 = 5$$

$$y + 3 = 5$$

$$y = 2$$

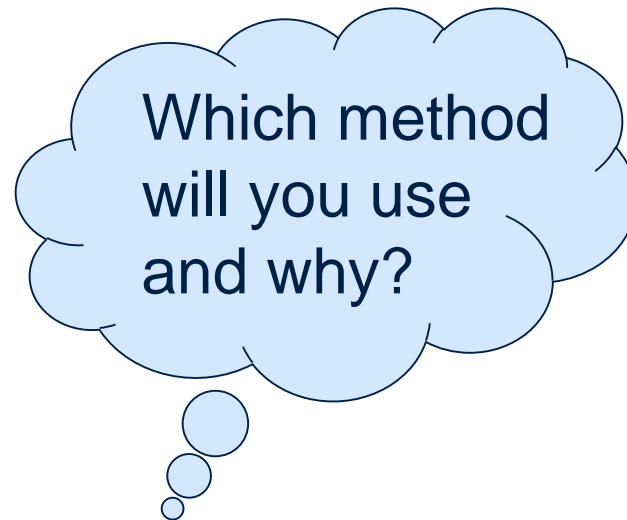
Which method is best and when?



Solve the following:

1. $2x + y = 7$
 $2x - y = 1$

2. $3x + 2y = 7$
 $3x + 5y = 4$



3. $y = 4x + 3$
 $3x + 2y = 28$

4. $4x + 3y = -4$
 $6x - 2y = 7$



Simultaneous Equations



Solutions on the next slide....



$$1. \begin{aligned} 2x + y &= 7 \\ 2x - y &= 1 \end{aligned}$$



Eliminate the y terms by adding the two equations

$$4x = 8$$

$$x = 2$$

Sub $x = 2$ into either equation to find y

$$2 \times 2 + y = 7$$

$$y = 3$$

$$2. \begin{aligned} 3x + 2y &= 7 \\ 3x + 5y &= 4 \end{aligned}$$



Eliminate the x terms by subtracting the second equation from the first

$$-3y = 3$$

$$y = -1$$

Sub $y = -1$ into either equation to find x

$$3x + 2 \times -1 = 7$$

$$x = 3$$

You can always check your answer is correct by substituting into the other equation and check it works in that one too



3. $y = 4x + 3$
 $3x + 2y = 28$



Substitute the first equation into the second

$$3x + 2(4x + 3) = 28$$

$$11x + 6 = 28$$

$$x = 2$$

Sub $x = 2$ into either equation to find y

$$y = 4 \times 2 + 3$$

$$y = 11$$

4. $4x + 3y = -4 \quad \times 2$
 $6x - 2y = 7 \quad \times 3$



Eliminate the y terms by adding the equations together

$$26x = 13$$

$$x = \frac{1}{2}$$

Subs $x = \frac{1}{2}$ into either equation to find x

$$4 \times \frac{1}{2} + 3y = -4$$

$$y = -2$$

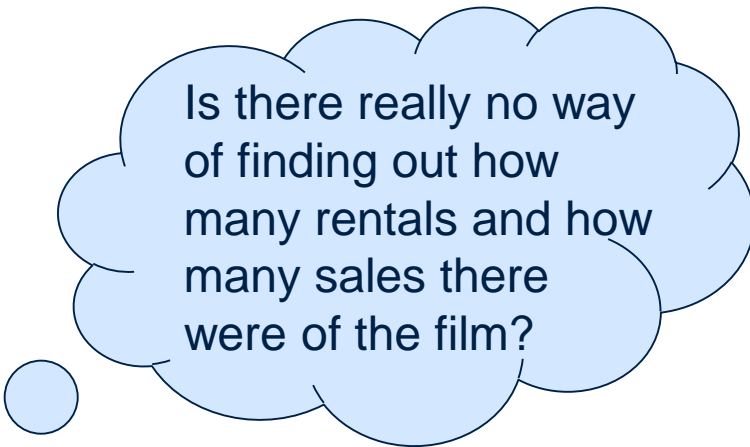
You can always check your answer is correct by substituting into the other equation and check it works in that one too



Maths movie makes millions!

“Our latest movie ‘Sum-body loves you’ has generated £15 million in online sales and rentals in the first week of it being released”
Simultaneous Studios said at the weekend.

We are unable to tell you how much of that total represents the £6 digital rental versus the £15 cost of purchasing the movie. But we do know there were 1 945 000 transactions overall.



Is there really no way of finding out how many rentals and how many sales there were of the film?

- Use what you have learnt so far to calculate how many individual rentals and sales there were of ‘Sum-body loves you’



Maths movie makes millions!

“Our latest movie ‘Sum-body loves you’ has generated £15 million in online sales and rentals in the first week of it being released”
Simultaneous Studios said at the weekend.

We are unable to tell you how much of that total represents the £6 digital rental versus the £15 cost of purchasing the movie. But we do know there were 1 945 000 transactions overall.

Let:

x be the number of rentals

y be the number of sales

then

$$x + y = 1\,945\,000$$

and

$$6x + 15y = 15\,000\,000$$

Multiply the first equation by 6 to get:

$$6x + 6y = 11\,670\,000$$

Eliminate x by subtracting this new equation from the second equation:

$$9y = 3\,330\,000$$

So $y = 370\,000$

and $x = 1\,575\,000$



There are two taxi companies



Initial Charge: $£x$
then
 $£1$ per mile



Initial Charge: $£2x$
then
80p per mile

They both charge $£12$ for a journey of the same distance.

- What is the distance?
- What is the value of x ?



There are two taxi companies



Initial Charge: $\pounds x$
then
 $\pounds 1$ per mile



Initial Charge: $\pounds 2x$
then
80p per mile

They both charge $\pounds 12$ for a journey of the same distance.

- What is the distance?
- What is the value of x ?

Let:

number of miles = n

then

$$x + n = 12$$

and

$$2x + 0.8n = 12$$

Multiply the first equation by 2 to get:

$$2x + 2n = 24$$

Eliminate x by subtracting the second equation from it

$$1.2n = 12$$

$$n = \frac{12}{1.2}$$

$$n = 10$$

$$x = 2$$

- The distance is 12 miles
- The value of x is $\pounds 2$

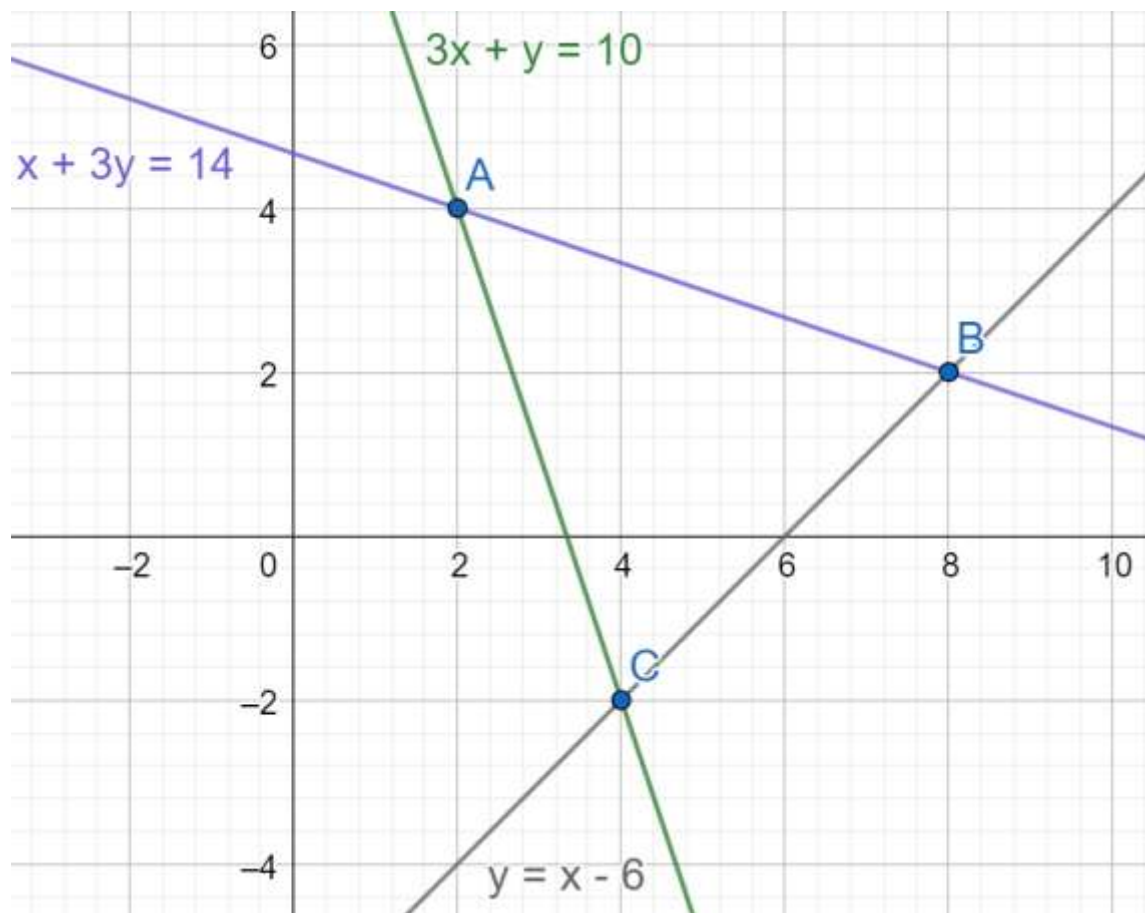


Use the graphs to solve these pairs of equations

1. $3x + y = 10$
 $x + 3y = 14$

2. $y = x - 6$
 $3x + y = 10$

3. $x + 3y = 14$
 $y = x - 6$

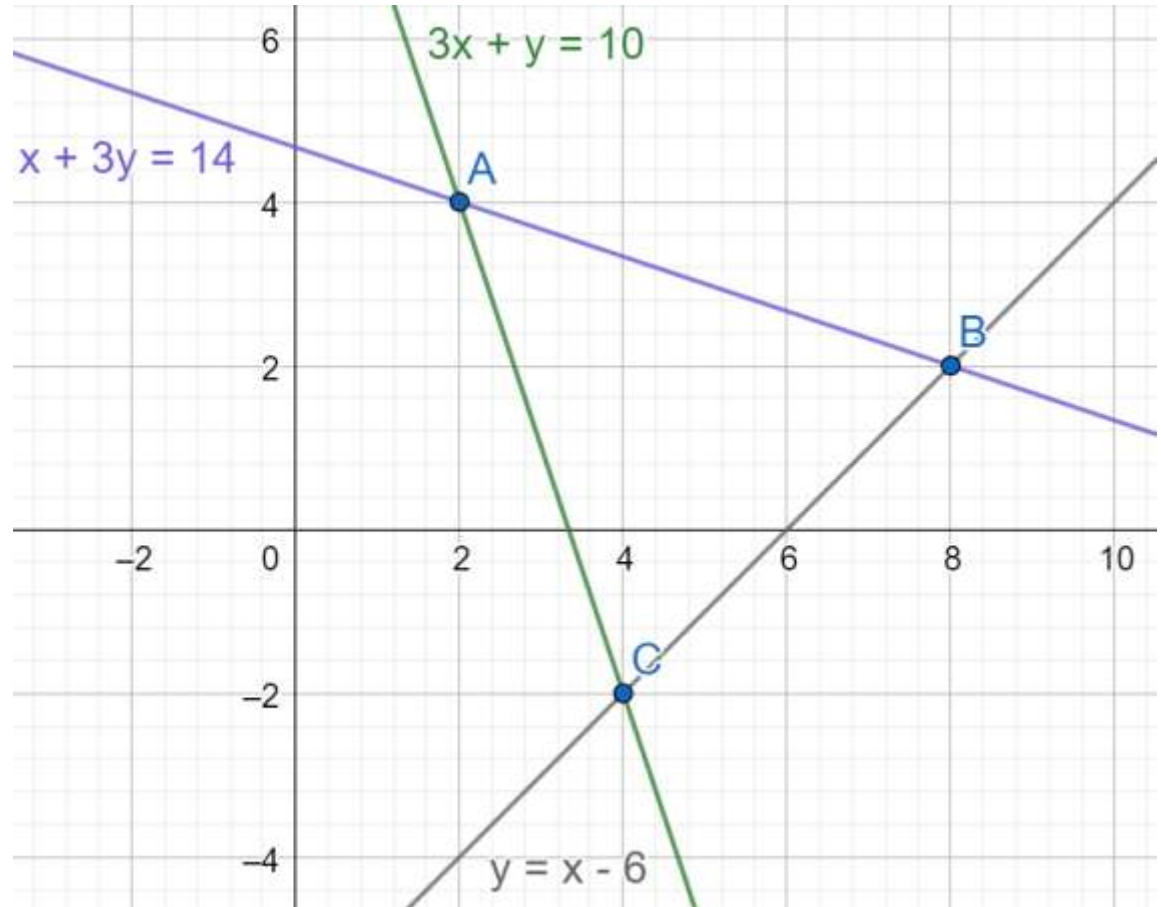




1. $3x + y = 10$
 $x + 3y = 14$
 Point A (2,4)
 $x = 2, y = 4$

2. $y = x - 6$
 $3x + y = 10$
 Point C (4,-2)
 $x = 4, y = -2$

3. $x + 3y = 14$
 $y = x - 6$
 Point B (8,2)
 $x = 8, y = 2$



When we have two equations, in x and y , the solution represents the point where the two lines meet.



Can you explain algebraically why there are no solutions to the simultaneous equations

$$y = 2x + 7$$
$$2y - 4x = 16$$

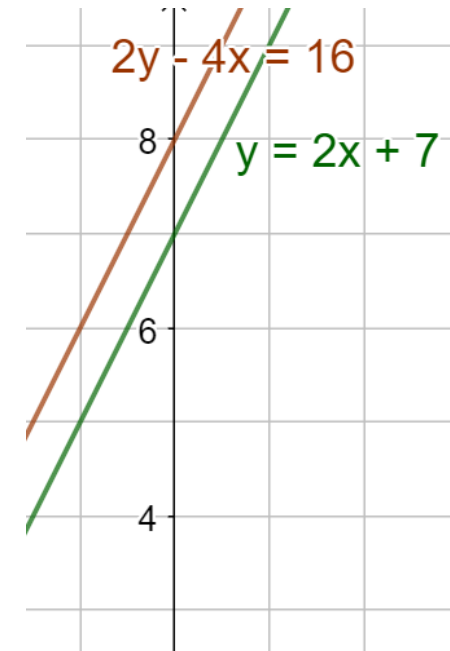


Can you explain algebraically why there are no solutions to the simultaneous equations

$$y = 2x + 7$$

$$2y - 4x = 16$$

- Rearrange the second equation
- So $y = 2x + 8$
- Both graphs have a gradient of 2.
- The lines are parallel
- Therefore they will never meet.



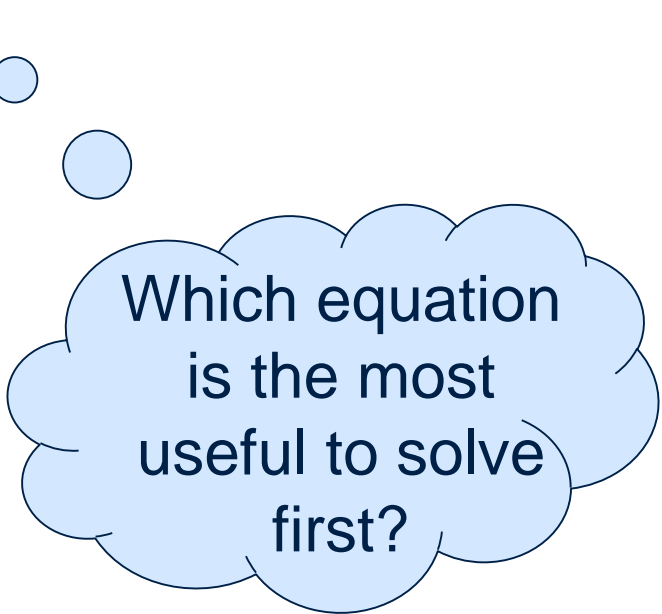


Solve

$$5x + 3y + z = 24$$

$$4y + 2z = 16$$

$$3z = 18$$



Which equation
is the most
useful to solve
first?



$$5x + 3y + z = 24$$

$$4y + 2z = 16$$

$$3z = 18$$

What would a picture (or graphs) of this situation look like?

- Dividing the third equation by 3 gives

$$z = 6$$

- Substituting $z = 6$ into the second equation gives

$$4y + 12 = 16$$

$$\text{So } y = 1$$

- Now, substitute $z = 6$ and $y = 1$ into the first equation to get

$$5x + 3 + 6 = 24, \text{ so } x = 3$$

- So the solution is $x = 3, y = 1, z = 6$



x , y and z satisfy

$$x + y + 3z = 121$$

$$x + 3y + z = 678$$

$$3x + y + z = 356$$

Find the mean of x, y, z , without using a calculator



x , y and z satisfy

$$x + y + 3z = 121$$

$$x + 3y + z = 678$$

$$3x + y + z = 356$$

Find the mean of x, y, z , without using a calculator

- Write an expression for the mean of x, y, z
- Do you need to find x, y, z separately to find the mean?



x, y, z satisfy

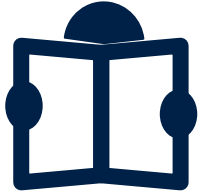
$$x + y + 3z = 121$$

$$x + 3y + z = 678$$

$$3x + y + z = 356$$

Find the mean of x, y, z , without using a calculator

- The mean of x, y, z is $\frac{x+y+z}{3}$
- Adding the three equations together gives
- $5x + 5y + 5z = 1155$
- Dividing by 5 we get
- $x + y + z = 231$
- Finally divide by 3 to get the mean $= \frac{x+y+z}{3} = 77$



Read how solving linear equations is an important part of many jobs – including those involving computer graphics, economics and genetics.



Discover the type of maths that is used when making blockbuster movies and how to do it.



Watch this animated history of operational research about its origins in the first and second world wars - when maths was used not only to improve operations but to save lives!

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