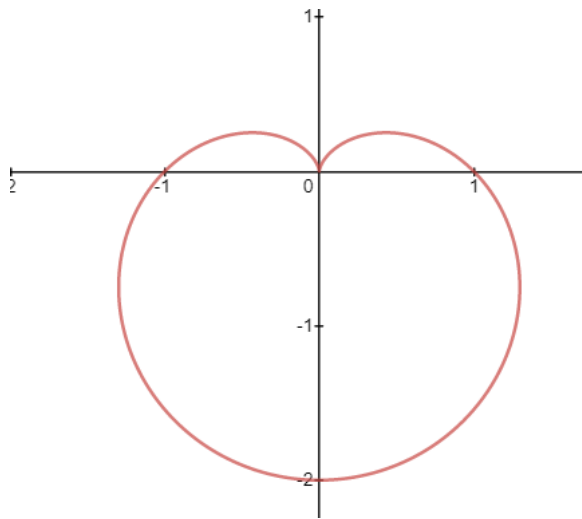


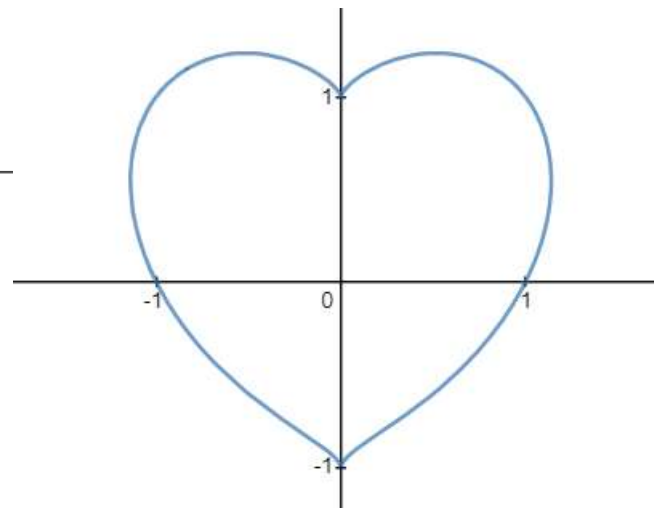


**Advanced Mathematics
Support Programme®**

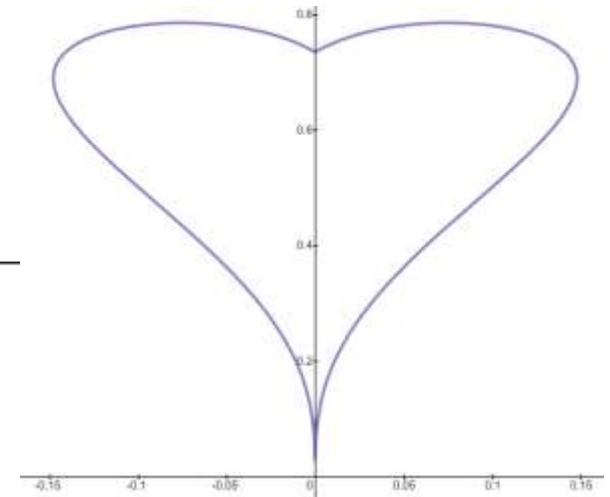
- Being able to express equations in different forms gives us different information
- Later we'll be looking at information needed to sketch graphs
- If you continue your maths studies to A Level Further Maths, you will draw graphs such as these



$$r = 1 - \sin\theta$$



$$(x^2 + y^2 - 1)^3 - x^2 y^3 = 0$$



$$x = \sin(t) \cos(t) \log(|t|)$$

$$y = |t|^{0.3} \sqrt{\cos(t)}$$



1. The equation of a line is given as
- $$3y + 4x - 2 = 0.$$
- What is the gradient of the line?

2. A rectangle has area A , length y and width $x - 2$. Write an expression for the length of the rectangle, y , in terms of A and x

3. Make x the subject of:
- $$ax - y = z + bx$$

4. Make b the subject of:
- $$5(b - p) = 2(b + 3)$$

5. John says the first step to rearranging $\frac{x-a}{f} = 3g$ is to add a to $3g$. Is he right? Explain your answer.

6. Make a the subject of
- $$5(a - t) = 3(a + x)$$

7. Make x the subject of
- $$ay + x = 4x + xb$$

8. Make x the subject of
- $$2\pi\sqrt{x + t} = 4$$



Further Factorising 1



Solutions on the next slide....



1. The equation of a line is given as

$$3y + 4x - 2 = 0.$$

What is the gradient of the line?



$$3y = -4x + 2$$

$$y = -\frac{4}{3}x + \frac{2}{3}$$

$$\text{gradient} = -\frac{4}{3}$$

2. A rectangle has area A , length y and width $x - 2$. Write an expression for the length of the rectangle, y , in terms of A and x



$$A = y(x - 2)$$

$$y = \frac{A}{x - 2}$$

3. Make x the subject of:

$$ax - y = z + bx$$



$$ax - bx = z + y$$

$$x(a - b) = z + y$$

$$x = \frac{z + y}{a - b}$$

4. Make b the subject of:

$$5(b - p) = 2(bx + 3)$$



$$5b - 5p = 2bx + 6$$

$$5b - 2bx = 6 + 5p$$

$$b(5 - 2x) = 6 + 5p$$

$$b = \frac{6 + 5p}{5 - 2x}$$



5. John says the first step to rearranging $\frac{x-a}{f} = 3g$ is to add a to $3g$. Is he right? Explain your answer.



No, the first step is to multiply by f

6. Make a the subject of
 $5(a - t) = 3(a + x)$



$$\begin{aligned} 5a - 5t &= 3a + 3x \\ 5a - 3a &= 3x + 5t \\ 2a &= 3x + 5t \\ a &= \frac{3x + 5t}{2} \end{aligned}$$

7. Make x the subject of
 $ay + x = 4x + xb$



$$\begin{aligned} ay &= 3x + xb \\ x(3 + b) &= ay \\ x &= \frac{ay}{3 + b} \end{aligned}$$

8. Make x the subject of
 $2\pi\sqrt{x+t} = 4$



$$\begin{aligned} \sqrt{x+t} &= \frac{4}{2\pi} \\ \sqrt{x+t} &= \frac{2}{\pi} \\ x+t &= \frac{4}{\pi^2} \quad x = \frac{4}{\pi^2} - t \end{aligned}$$



1. Make y the subject of

$$xy + 6 = 7 - ky$$

2. Find an expression for the area of a rectangle with length, $(y - x)$ and width, $(x - 2)$

3. Rewrite your expression in Q2 to have y expressed in terms of A and x

4. Make y the subject of $\frac{4}{y} + 1 = 2x$

5. Displacement can be expressed as

$$s = ut + \frac{1}{2}at^2$$

Express a in terms of s, u and t

6. Make y the subject of $\sqrt{by^2 - x} = D$

7. The area of a trapezium has formula

$$A = \frac{1}{2} \left(\frac{a+b}{h} \right)$$

Express h in terms of A, a and b

8. Make t the subject $b(t + a) = x(t + b)$



Further Factorising 2



Solutions on the next slide....



1. Make y the subject of

$$xy + 6 = 7 - ky$$



$$\begin{aligned} xy + ky &= 1 \\ y(x + k) &= 1 \\ y &= \frac{1}{x + k} \end{aligned}$$

2. Find an expression for the area of a rectangle with length, $(y - x)$ and width, $(x - 2)$



$$\begin{aligned} \text{Area} &= (y - x)(x - 2) \\ A &= xy - x^2 - 2y + 2x \end{aligned}$$

3. Rewrite your expression in Q2 to have y expressed in terms of A and x



$$\begin{aligned} A &= xy - x^2 - 2y + 2x \\ 2y - xy &= 2x - x^2 - A \\ y(2 - x) &= 2x - x^2 - A \\ y &= \frac{2x - x^2 - A}{2 - x} \end{aligned}$$

4. Make y the subject of $\frac{4}{y} + 1 = 2x$



$$\begin{aligned} \frac{4}{y} &= 2x - 1 \\ y(2x - 1) &= 4 \\ y &= \frac{4}{2x - 1} \end{aligned}$$



5. Displacement can be expressed as →

$$s = ut + \frac{1}{2}at^2$$

Express a in terms of s, u and t

$$\begin{aligned} \frac{1}{2}at^2 &= s - ut \\ at^2 &= 2s - 2ut \\ a &= \frac{2s - 2ut}{t^2} \end{aligned}$$

Solution

6. Make y the subject of $\sqrt{by^2 - x} = D$ →

$$\begin{aligned} by^2 - x &= D^2 \\ by^2 &= D^2 + x \\ y^2 &= \frac{(D^2 + x)}{b} \\ y &= \pm \sqrt{\frac{D^2 + x}{b}} \end{aligned}$$

7. The area of a trapezium has formula →

$$A = \frac{1}{2} \left(\frac{a+b}{h} \right)$$

Express h in terms of A, a and b

$$\begin{aligned} 2hA &= a + b \\ h &= \frac{a + b}{2A} \end{aligned}$$

8. Make t the subject $b(t + a) = x(t + b)$ →

$$\begin{aligned} bt + ba &= xt + xb \\ bt - xt &= xb - ba \\ t(b - x) &= xb - ba \\ t &= \frac{xb - ba}{b - x} \end{aligned}$$



Sort the expressions below in to 4 sets of 4 equivalent expressions.

$x^2 - 25$	$2x^2 - 2$
$(x + 5)(x + 6) - x - 55$	$(x + 5)(x - 5)$
$2(x^2 - 1)$	$(x + 5)^2 - 10x - 50$
$2(x + 3)(x - 1)$	$2(x + 1)(x - 1)$
$(x + 5)^2 - 50$	$2(x + 2)^2 - 4x - 14$
$2x^2 + 4x - 6$	$(x + 5)(x - 5) + 10x$
$2(x + 1)^2 - 8$	$(x - 5)(x + 6) - x + 5$
$x^2 + 10x - 25$	$2(x + 1)^2 - 4(x - 1)$

Equivalent quadratics



Solutions on the next slide....



Sort the expressions below in to 4 sets of 4 equivalent expressions.

$x^2 - 25$	$2x^2 - 2$
$(x + 5)(x + 6) - x - 55$	$(x + 5)(x - 5)$
$2(x^2 - 1)$	$(x + 5)^2 - 10x - 50$
$2(x + 3)(x - 1)$	$2(x + 1)(x - 1)$
$(x + 5)^2 - 50$	$2(x + 2)^2 - 4x - 14$
$2x^2 + 4x - 6$	$(x + 5)(x - 5) + 10x$
$2(x + 1)^2 - 8$	$(x - 5)(x + 6) - x + 5$
$x^2 + 10x - 25$	$2(x + 1)^2 - 4(x - 1)$



- Take two positive values greater than 1
- Find the mean of the two values
- Square it

Then

- Take the same two values
- Square them
- Find the mean of the squares

Which value is greater?

Is this always true?

Can you prove it?



- Take two positive values greater than 1
- Find the mean of the two values
- Square it

Then

- Take the same two values
- Square them
- Find the mean of the squares

Which value is greater?

Is this always true?

Can you prove it?

- Try out several examples
- Is one expression always bigger than the other?
- Next try using x and y instead.
- If you subtract one expression from the other, can you work out if it's positive or negative?

Mean squares



Follow the [link](#) to the solutions



Problem 1:

Mrs Gryce was asked to calculate 18×12 by Mr Lo who had forgotten his calculator and was doing some marking.

Mrs Gryce quickly responded

“Well, that’s just $15^2 - 9$ which is 216”

Mr Lo was amazed.

- How did she know so quickly what the answer was?



Problem 2:

- Use the fact that $3 \times 4 = 12$
- Can you quickly work out a value for $(3.5)^2$?

Can you see a connection between the previous question and this one?

Difference of numeric squares



Follow the [link](#) to the solutions



We've all used the Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- But where does it come from?
- Can you prove why the quadratic formula works?

The next activity is all about doing just that!



Rearrange these steps in order to prove the quadratic formula

$$ax^2 + bx + c = 0 \longrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\left(x + \frac{b}{2a}\right) = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} = -\frac{c}{a}$$

$$ax^2 + bx = -c$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$\left(x + \frac{b}{2a}\right) = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$



Match the steps below with the algebra on the previous slide for a slightly easier version

Step 1: Subtract c from both sides

Step 2: Divide both sides by a

Step 3: Complete the square on the left hand side

Step 4: Add $\frac{b^2}{4a^2}$ to both sides

Step 5: Make the right hand side into a single expression

Step 6: Take the square root of both sides

Step 7: Simplify the denominator on the right hand side

Step 8: Subtract $\frac{b}{2a}$ from both sides

Step 9: You now have the quadratic formula!

For a challenge – just use the algebra!

The Quadratic Formula - Rearranging



Follow the [link](#) to the solutions



You should have come across an equation like this in your GCSE course:

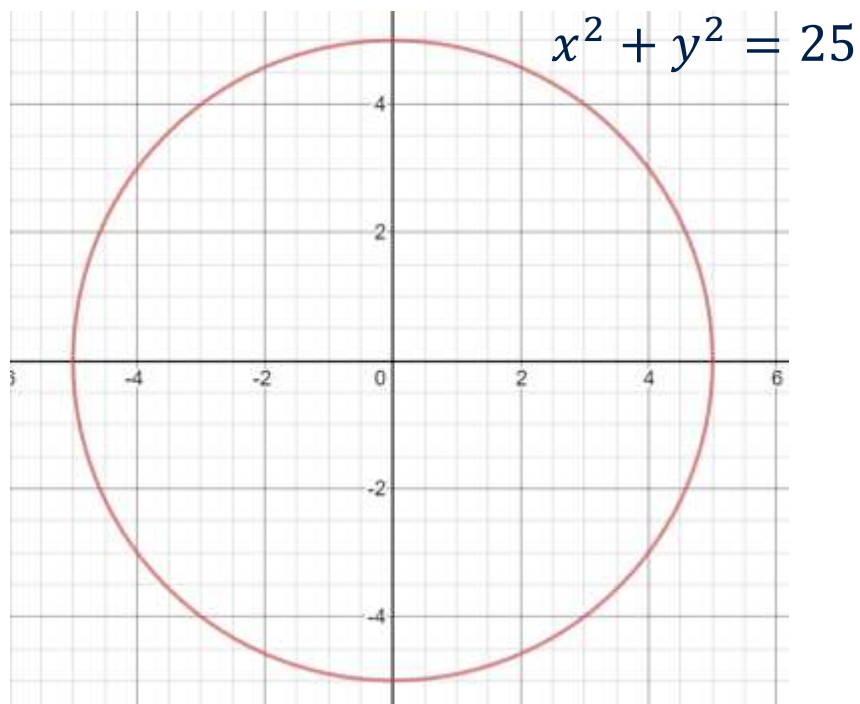
$$x^2 + y^2 = 25$$

- Can you remember what this would look like?
- Can you describe it?



$$x^2 + y^2 = 25$$

Represents a circle with centre (0,0) and radius 5



Generally the equation of a circle with centre (0,0) and radius r can be written as

$$x^2 + y^2 = r^2$$

But what happens if the centre is not (0,0) ?



Let's have a look at this equation

$$x^2 + 4x + y^2 - 6y = 12$$

We can rearrange this by completing the square separately for the x terms and y terms

$$x^2 + 4x = (x + 2)^2 - 4 \text{ and } y^2 - 6y = (y - 3)^2 - 9$$

So

$$x^2 + 4x + y^2 - 6y = 12$$

Can be written as

$$(x + 2)^2 - 4 + (y - 3)^2 - 9 = 12$$

$$(x + 2)^2 + (y - 3)^2 - 13 = 12$$

Collect like terms

$$(x + 2)^2 + (y - 3)^2 = 25$$

Rearrange to become

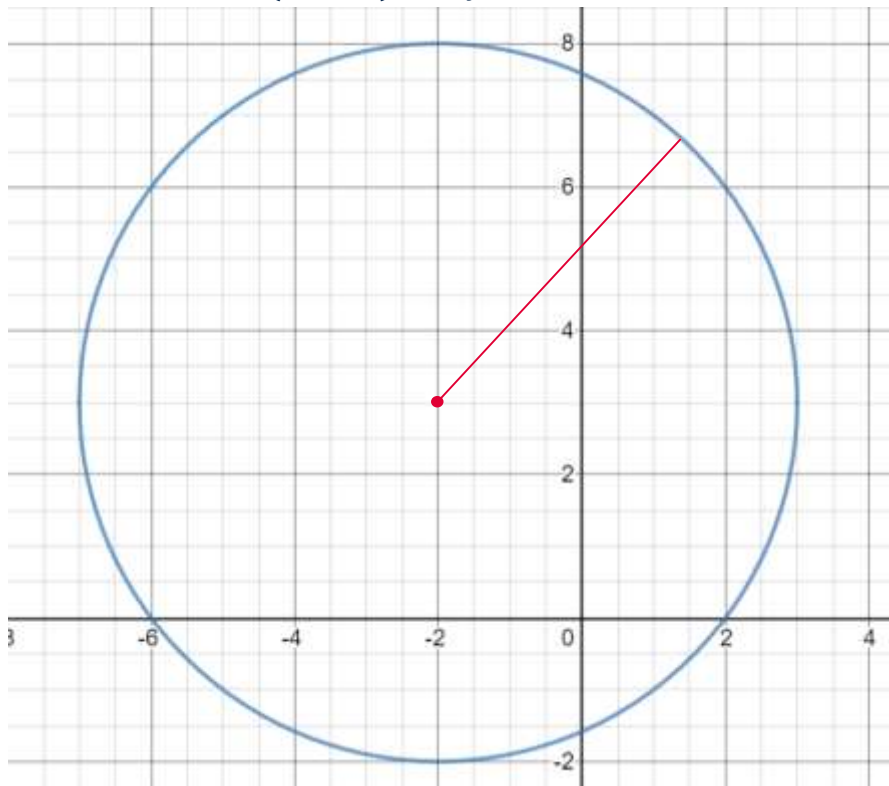
What do you think this equation represents?



$$(x + 2)^2 + (y - 3)^2 = 25$$

Represents a circle with
Centre (-2,3)
Radius 5

$$(x + 2)^2 + (y - 3)^2 = 25$$



If we expand this equation

$$\begin{aligned} (x + 2)^2 + (y - 3)^2 &= 25 \\ x^2 + 4x + 4 + y^2 - 6y + 9 &= 25 \\ x^2 + 4x + y^2 - 6y + 13 &= 25 \\ x^2 + 4x + y^2 - 6y &= 12 \end{aligned}$$

We return it to the original form



- Can you find the centre and radii of these circles by rearranging into the form

$$(x + a)^2 + (y - b)^2 = r^2$$

$$x^2 - 8x + y^2 - 2y = 19$$

$$x^2 + 6x + y^2 - 10y = 8$$

Equations of circles



Follow the [link](#) to the solutions



Discover how using a graphing app such as Desmos or GeoGebra can help you gain insight into circles, tangents and graphs in general. Gain skills useful for A level maths.



Watch a TED talk from Dr Hannah Fry which tries to answer the question “Is life too complex?” You will see that you can actually write equations that model human behaviour!

Contact the AMSP



01225 716 492



admin@amsp.org.uk



amsp.org.uk



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