Advanced Mathematics Support Programme®

Did you know?

I have picked two numbers that multiply to make zero.

What can you say about my numbers?

At least one of them must be zero

This is useful when using factorising to solve equations.

If $a \times b = 0$, then either $a = 0$ or $b = 0$ (or both!)

Historically zero wasn't accepted as a number until relatively recently!

Solving with Quadratics 1

Solve the following

1. $x^2 = 16$

2. $x^2 - 16x = 0$

3. $(x+1)(2x-3) = 0$

4. $x^2 - 3x + 2 = 0$

5. $(2x-5)(4x+3) = 0$

6. $3x^2 + 14x - 5 = 0$

7. $(x+3)^2 = 25$

8. $\frac{3}{x} + \frac{4}{x+1} = 10$

Solving with Quadratics 1

Solutions on the next slide….

Quadratics 1 Solutions

1. $x^2 = 16$ → $x = \pm\sqrt{16}$

   $x = \pm 4$

2. $x^2 - 16x = 0$ → $x(x - 16) = 0$

   $x = 0 \text{ or } x = 16$

3. $(x + 1)(2x - 3) = 0$ → $x + 1 = 0 \text{ or } 2x - 3 = 0$

   $x = -1 \text{ or } x = \frac{3}{2}$

4. $x^2 - 3x + 2 = 0$ → $(x - 2)(x - 1) = 0$

   $x = 2 \text{ or } x = 1$

Unsure about any of these? Search Solving quadratic equations. Next try Quadratics 2….

Quadratics 1 Solutions

5. $(2x-5)(4x+3)=0$

$$2x-5=0 \ or \ 4x+3=0$$
$$2x=5 \ or \ 4x=-3$$
$$x=\frac{5}{2} \ or \ x=-\frac{3}{4}$$

6. $3x^2+14x-5=0$

$$(3x-1)(x+5)=0$$
$$3x-1=0 \ or \ x+5=0$$
$$3x=1 \ or \ x=-5$$
$$x=\frac{1}{3} \ or \ x=-5$$

7. $(x+3)^2=25$

$$x+3=\pm\sqrt{25}$$
$$x+3=\pm 5$$
$$x=2 \ or \ x=-8$$

8. $\frac{3}{x}+\frac{4}{x-1}=10$

$$\frac{3(x-1)+4x}{x(x-1)}=10$$
$$3x-3+4x=10x(x-1)$$
$$7x-3=10x^2-10\text{x}$$
$$10x^2-17x+3=0$$
$$(2x-3)(5x-1)=0$$
$$x=\frac{3}{2} \ or \ x=\frac{1}{5}$$

Solving with Quadratics 2

Solve the following

1. $x^2 - 4x - 12 = 0$

2. $x^2 - x = 6$

3. $2x^2 - 11x + 12 = 0$

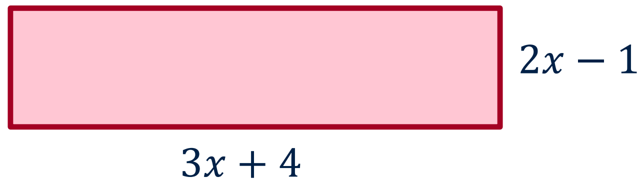
4. $6x^2 + x - 12 = 0$

5. $3 + 2x - x^2 = 0$

6. $x^2 - 4x - 1 = 0$

7. $\dfrac{8}{x+2} - \dfrac{14}{x-3} = 9$

8. The area of this rectangle is $30m^2$



a) Show that $6x^2 + 5x - 34 = 0$
b) Find any possible values for x

Solving with Quadratics 2

Solutions on the next slide….

Solutions Quadratics 2

1. $x^2 - 4x - 12 = 0$ →

$$(x-2)^2 - 4 - 12 = 0$$
$$(x-2)^2 = 16$$
$$x - 2 = \pm 4$$
$$x = 6 \; or \; x = -2$$

We have used completing the square. Factorising can also be used.

2. $x^2 - x = 6$ →

$$x^2 - x - 6 = 0$$
$$(x-3)(x+2) = 0$$
$$x = 3 \; or \; x = -2$$

3. $2x^2 - 11x + 12 = 0$ →

$$(2x-3)(x-4) = 0$$
$$2x - 3 = 0 \; or \; x - 4 = 0$$
$$2x = 3 \; or \; x = 4$$
$$x = \frac{3}{2} \; or \; x = 4$$

4. $6x^2 + x - 12 = 0$ →

$$(2x+3)(3x-4) = 0$$
$$2x + 3 = 0 \; or \; 3x - 4 = 0$$
$$2x = -3 \; or \; 3x = 4$$
$$x = -\frac{3}{2} \; or \; x = \frac{4}{3}$$

Solutions Quadratics 2

5. $3 + 2x - x^2 = 0$

$$(3 - x)(1 + x) = 0$$
$$x = 3 \text{ or } x = -1$$

6. $x^2 - 4x - 1 = 0$

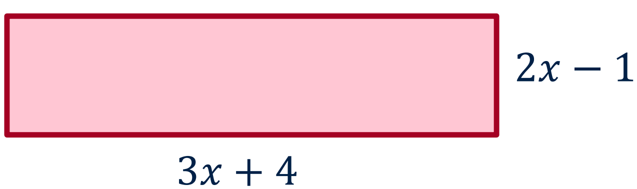
$$(x - 2)^2 - 4 - 1 = 0$$
$$(x - 2)^2 = 5$$
$$x - 2 = \pm\sqrt{5}$$
$$x = 2 \pm \sqrt{5}$$

We have used completing the square. The quadratic formula can also be used

7. $\dfrac{8}{x + 2} - \dfrac{14}{x - 3} = 9$

$$\frac{8(x - 3) - 14(x + 2)}{(x + 2)(x - 3)} = 9$$
$$8x - 24 - 14x - 28 = 9(x + 2)(x - 3)$$
$$-6x - 52 = 9x^2 - 9x - 54$$
$$9x^2 - 3x - 2 = 0$$
$$(3x + 1)(3x - 2) = 0$$
$$x = -\frac{1}{3} \text{ or } x = \frac{2}{3}$$

8. The area of this rectangle is $30m^2$



a) Show that $6x^2 + 5x - 34 = 0$
b) Find any possible values for x

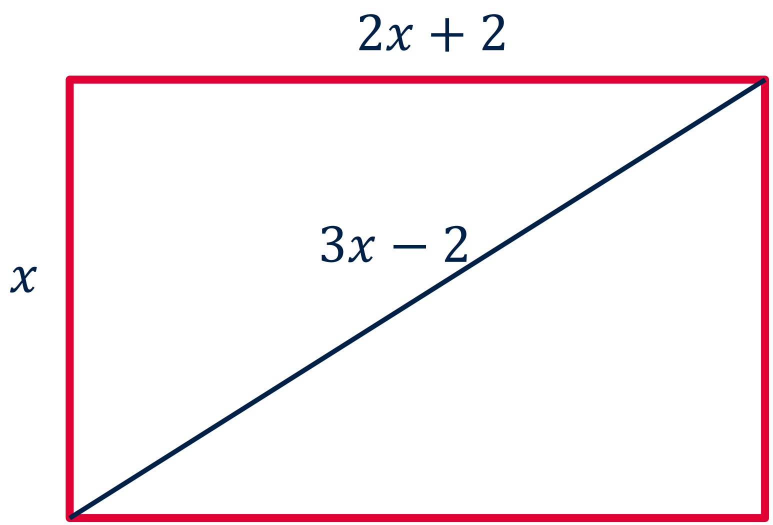
$$(2x - 1)(3x + 4) = 30$$
$$6x^2 + 5x - 4 = 30$$
$$6x^2 + 5x - 34 = 0$$
$$(6x + 17)(x - 2) = 0$$

$x = 2$ Note $x \neq -\dfrac{17}{6}$

Side lengths can't be negative

Quadthagoras

Find the length, width and diagonal of this rectangle

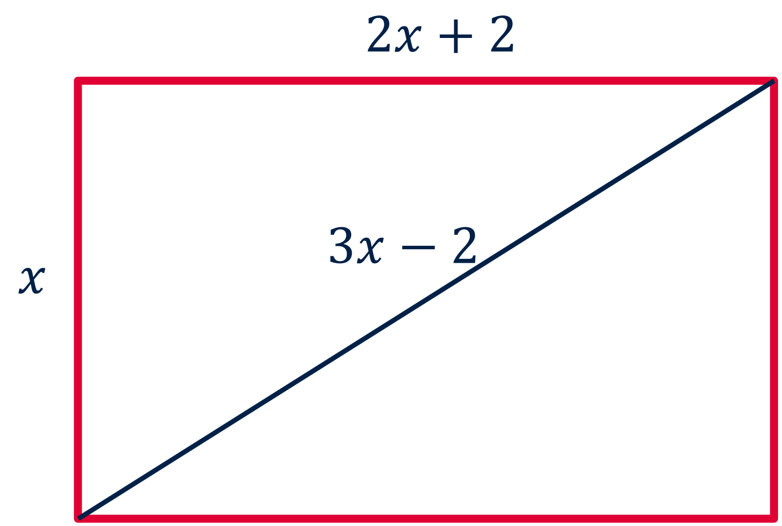


Quadthagoras

Solutions on the next slide….

Quadthagoras Solution

Find the length, width and diagonal of this rectangle



By Pythagoras' Theorem:

$$x^2 + (2x+2)^2 = (3x-2)^2$$
$$x^2 + 4x^2 + 8x + 4 = 9x^2 - 12x + 4$$
$$4x^2 - 20x = 0$$
$$4x(x-5) = 0$$
$$x = 0 \text{ or } x = 5$$

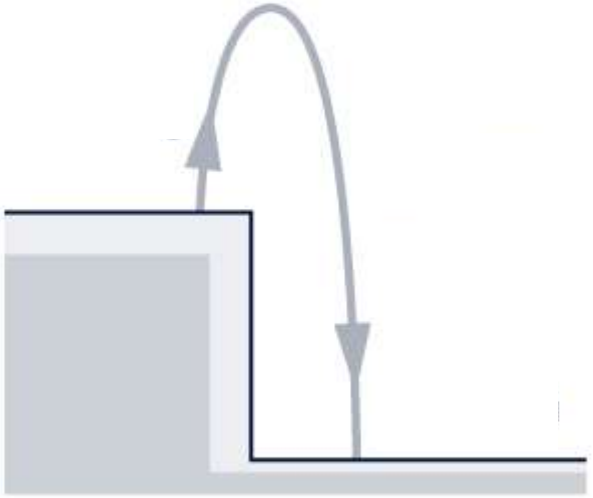
- As we are finding lengths, only $x = 5$ makes sense in this context.
- Therefore suitable lengths are 5, 12 and 13

Up in the air!

An object is launched from a cliff that is $58.8m$ high.
The speed of the object is 19.6 metres per second (m/s).

The equation for the object's height h above the ground at time t seconds after launch is $h = -4.9t^2 + 19.6t + 58.8$ where h is in metres.

- When does the object strike the ground?



Up in the air!

Solutions on the next slide….

Up in the air Solution

An object is launched from a cliff that is $58.8m$ high.
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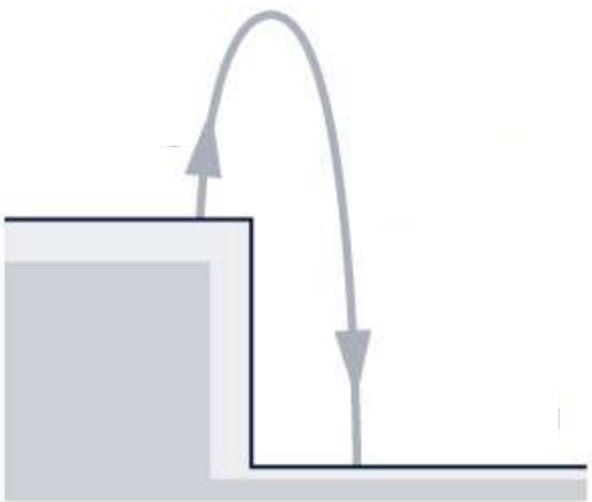


The object will hit the ground when $h = 0$
So we need to solve $0 = -4.9t^2 + 19.6t + 58.8$

Tip: rearrange to make t^2 positive

$$4.9t^2 - 19.6t - 58.8 = 0$$

Tip: 4.9 is a factor of 19.6 and 58.8

There are other methods you can use to solve this equation

$$t^2 - 4t - 12 = 0$$

$$(t-6)(t+2) = 0$$

$t = 6$ or $t = -2$ the object strikes the ground after 6 seconds

The answer is a positive as it represents the time after launch

Which Way?

In the skills check you saw how we can solve quadratic equations by factorising or completing the square.

We can also use the quadratic formula, for a quadratic $ax^2 + bx + c = 0$ the solutions are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Try solving $x^2 + 4x - 21 = 0$ using each of the three methods.

Try solving $3x^2 + 4x - 2 = 0$ using each of the three methods.

Which Way?

Solutions on the next slide….

Which Way? Solutions

Solve by

$$x^2 + 4x - 21 = 0$$

Factorising

$$(x+7)(x-3) = 0$$

$x = -7$ or $x = 3$

Completing the square

$$(x+2)^2 - 4 - 21 = 0$$
$$(x+2)^2 = 25$$
$$(x+2) = \pm 5$$

$x = -7$ or $x = 3$

Quadratic Formula

$$a = 1, \mathrm{b} = 4, \mathrm{c} = -21$$
$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-21)}}{2(1)}$$
$$= \frac{-4 \pm \sqrt{16 + 84}}{2}$$
$$= \frac{-4 \pm \sqrt{100}}{2}$$
$$= \frac{-4 \pm 10}{2}$$
$$= -2 \pm 5$$

$x = -7$ or $x = 3$

Which of the methods would be your first choice here?

Was it different when you tried to solve $3x^2 + 4x - 2 = 0$?

Which Way? Solutions

Solve by

$$3x^2 + 4x - 2 = 0$$

Factorising

It doesn't factorise

Horrible!

Completing the square

$$3[x^2 + \frac{4}{3}x - \frac{2}{3}] = 0$$

$$3[\left(x + \frac{2}{3}\right)^2 - \frac{4}{9} - \frac{2}{3}] = 0$$

$$3[\left(x + \frac{2}{3}\right)^2 - \frac{10}{9}] = 0$$

$$3\left(x + \frac{2}{3}\right)^2 - \frac{30}{9} = 0$$

$$3\left(x + \frac{2}{3}\right)^2 = \frac{30}{9}$$

$$\left(x + \frac{2}{3}\right)^2 = \frac{10}{9}$$

$$\left(x + \frac{2}{3}\right) = \pm\sqrt{\frac{10}{9}}$$

$x = -\frac{2}{3} + \frac{\sqrt{10}}{3}$ or $x = -\frac{2}{3} - \frac{\sqrt{10}}{3}$

Quadratic Formula

$$a = 3, \mathrm{b} = 4, \mathrm{c} = -2$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(3)(-2)}}{2(3)}$$

$$= \frac{-4 \pm \sqrt{16 + 24}}{6}$$

$$= \frac{-4 \pm \sqrt{40}}{6}$$

$$= -\frac{2}{3} \pm \frac{\sqrt{10}}{3}$$

$x = -\frac{2}{3} + \frac{\sqrt{10}}{3}$

or

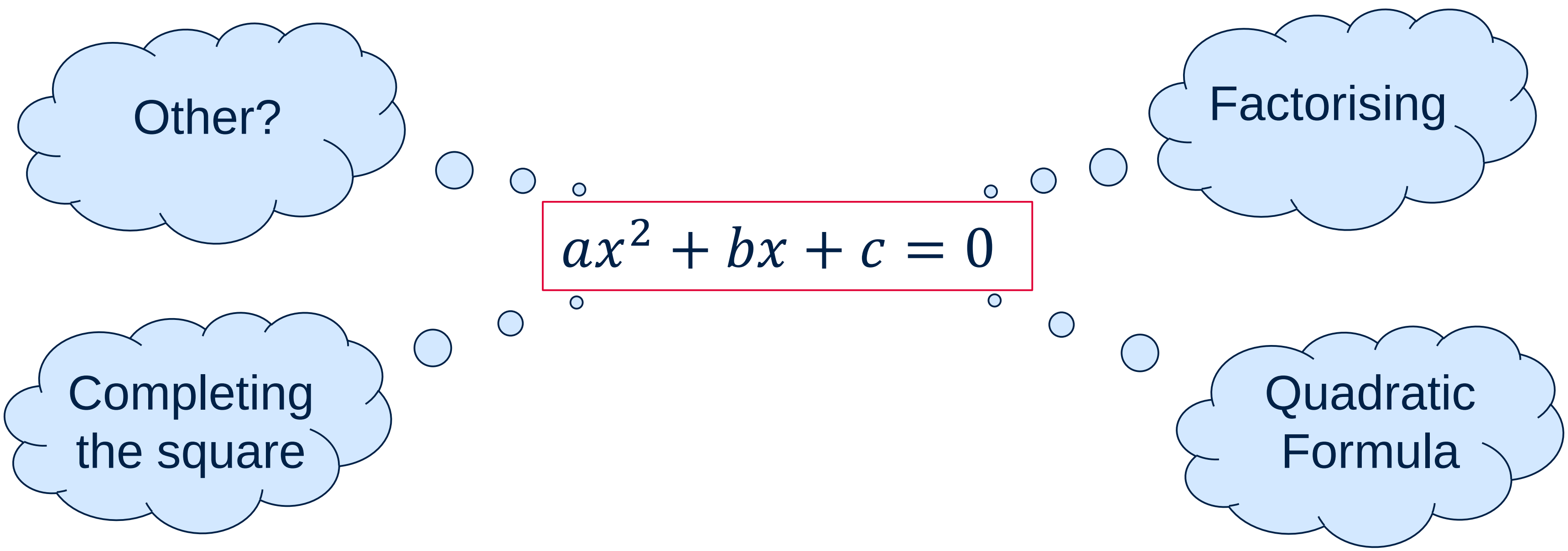
$x = -\frac{2}{3} - \frac{\sqrt{10}}{3}$

Which Way Now?

There is not always one best way to solve a quadratic.

Some methods are better than others for different equations

How can you spot which is the right method for each equation?

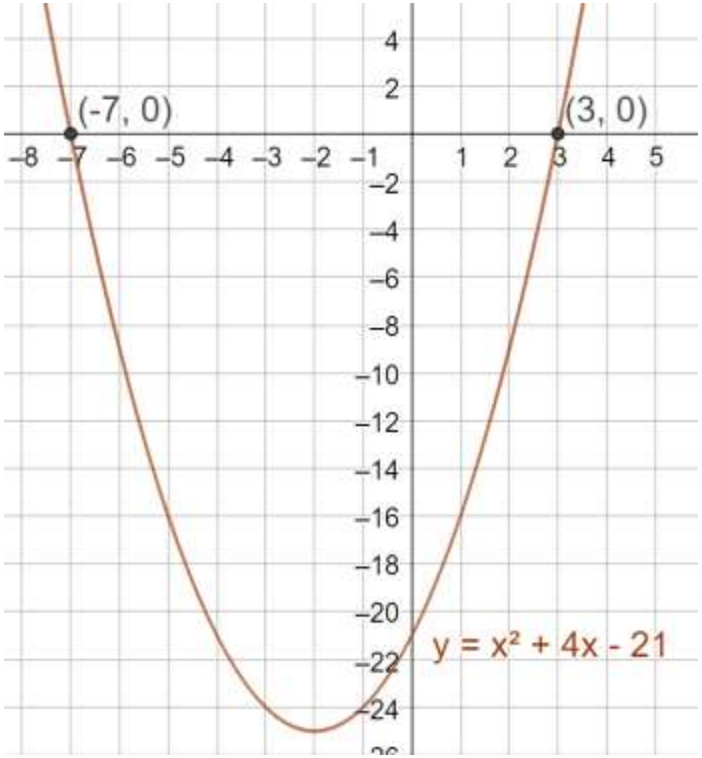


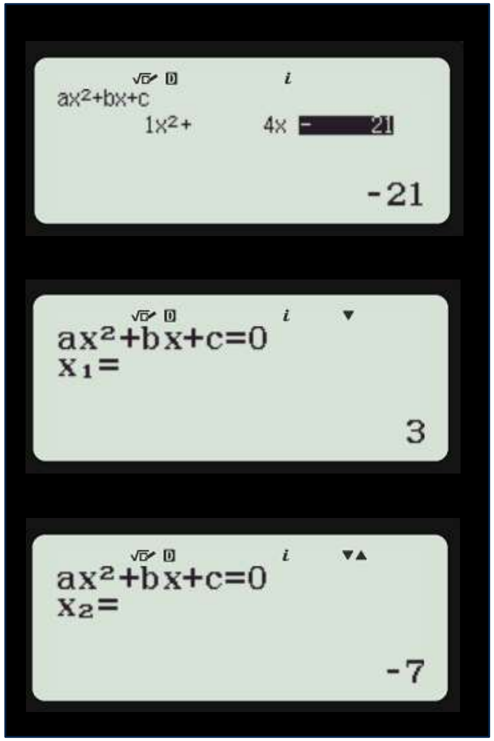
Try this activity to improve your skills by sorting quadratic equations.

Another Way?

And of course there are the methods of solving using graphs and/or your calculator

$$x^2 + 4x - 21 = 0$$





Using Graphs

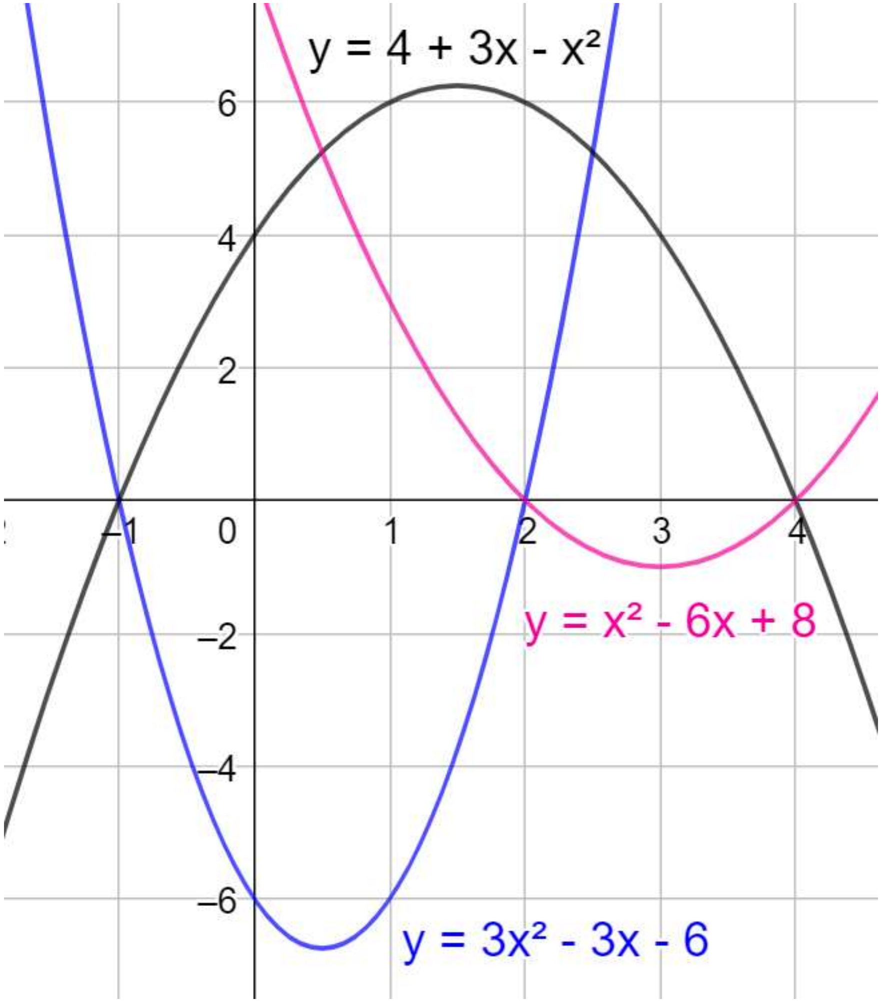
Use the graphs to solve

$$4 + 3x - x^2 = 0$$

$$x^2 - 6x + 8 = 0$$

$$3x^2 - 3x - 6 = 0$$

$$4 + 3x - x^2 = 4$$



Using Graphs

Solutions on the next slide….

Using Graphs Solution

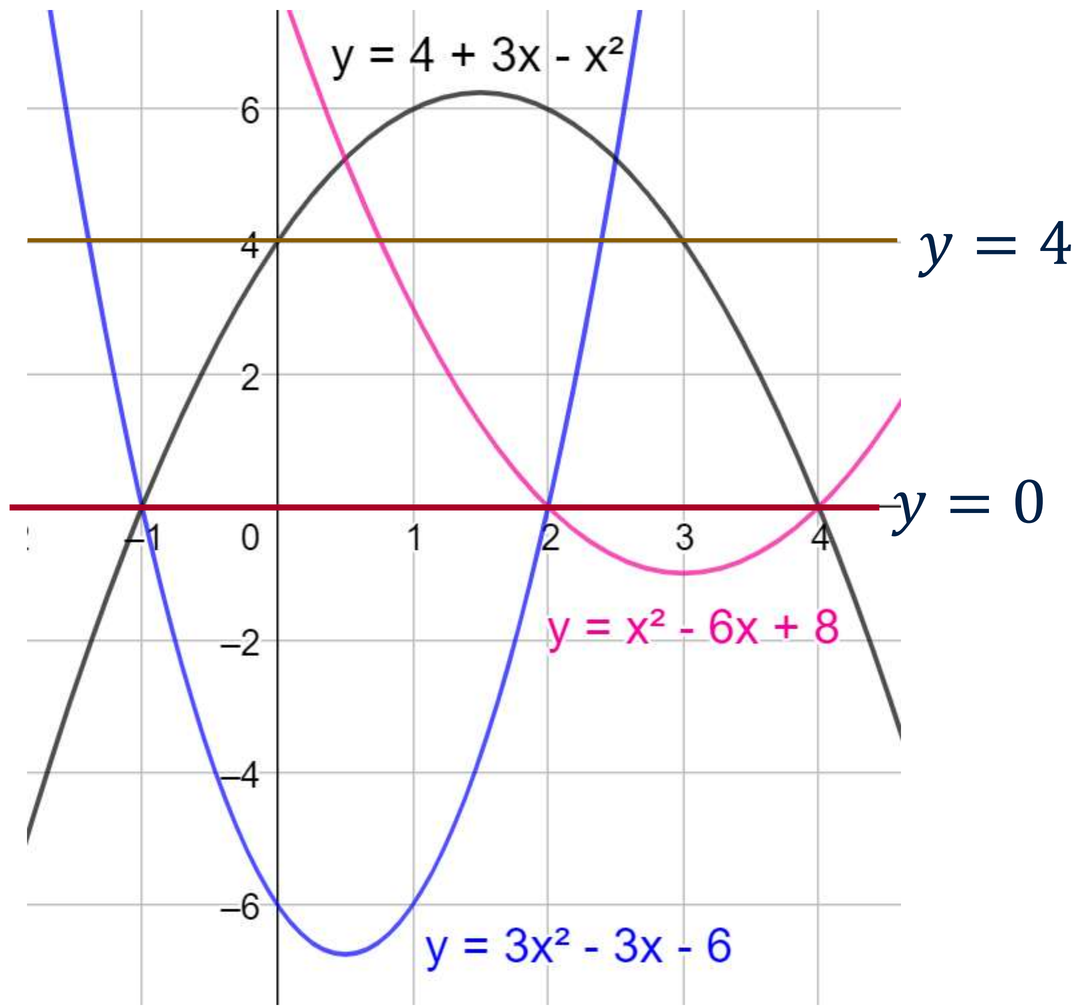
Use the graphs to solve

$$4 + 3x - x^2 = 0$$
$$x = -1 \; or \; x = 4$$

$$x^2 - 6x + 8 = 0$$
$$x = 2 \; or \; x = 4$$

$$3x^2 - 3x - 6 = 0$$
$$x = -1 \; or \; x = 2$$

$$4 + 3x - x^2 = 4$$
$$x = 0 \; or \; x = 3$$



Simultaneously

Solve these pairs of equations

1. $y = x^2 + 6x - 9$
   $y = 3x + 1$

2. $y = x^2 + 2x + 2$
   $y - 4x = 1$

Simultaneously

A rectangle has length $(a + b)$ and width $3a$.

The area is $60cm^2$ and perimeter is $32\ cm$.

Calculate, algebraically, the possible values for a and b.

In how many places does the line $y = 2x + 2$ intersect the circle $(x + 2)^2 + y^2 = 25$?

What are the co-ordinates of these intersections?

Simultaneously

Solutions on the next slide….

Simultaneously Solutions

1.

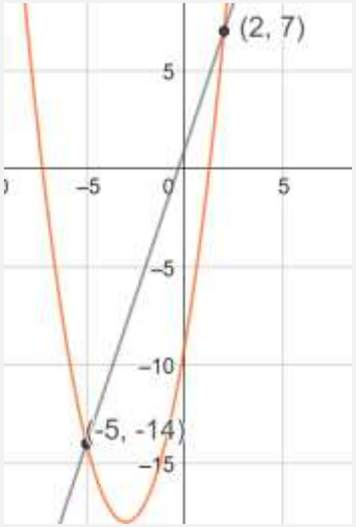
$$y = x^2 + 6x - 9$$
$$y = 3x + 1$$

Set equations equal to each other

$$x^2 + 6x - 9 = 3x + 1$$
$$x^2 + 3x - 10 = 0$$
$$(x + 5)(x - 2) = 0$$
$$x = 2 \ or \ -5$$

Substitute in to $y = 3x + 1$

$$x = 2, y = 7$$
$$x = -5, y = -14$$



2.

$$y = x^2 + 2x + 2$$
$$y - 4x = 1$$

Rearrange $y - 4x = 1$

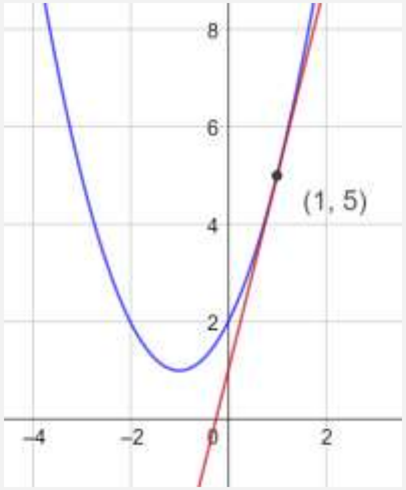
To $y = 4x + 1$

Set equations equal to each other

$$x^2 + 2x + 2 = 4x + 1$$
$$x^2 - 2x + 1 = 0$$
$$(x - 1)^2 = 0$$
$$x = 1$$

Substitute in to $y = 4x + 1$

$$x = 1, y = 2$$



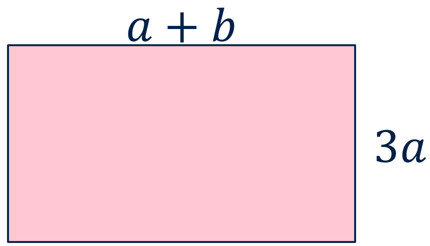
If you look at the graph you can see there is only one place where the line and curve meet – which is why there is only one solution. The straight line doesn't cross the curve but just touches. This is called a **Tangent**

Simultaneously Solutions

A rectangle has length $(a+b)$ and width $3a$.

The area is $60cm^2$ and perimeter is $32\ cm$.

Calculate, algebraically, the possible values for a and b.



Perimeter:

$$2(a+b+3a)=32$$
$$2(4a+b)=32$$
$$4a+b=16$$

Rearrange to get:

$$b=16-4a$$

Area:

$$3a(a+b)=60$$
$$3a^2+3ab-60=0$$

Substitute for b into the Area equation

$$3a^2+3a(16-4a)-60=0$$
$$-9a^2+48a-60=0$$

Rearrange to look nicer

$$9a^2-48a+60=0$$
$$(3a-10)(3a-6)=0$$

Can be solved by other methods too

$$a=\frac{10}{3}\ or\ a=2$$

Substitute back into $b=16-4a$

When $a=\frac{10}{3}$ $\quad b=\frac{8}{3}$ $\qquad$ When $a=2$ $\quad b=8$

Simultaneously solutions

In how many places does the line $y = 2x + 2$ intersect the circle $(x+2)^2 + y^2 = 25$?

What are the co-ordinates of these intersections?

Substitute for y into the second equation

$$y = 2x + 2$$
$$(x+2)^2 + y^2 = 25$$

$$(x+2)^2 + (2x+2)^2 = 25$$
$$(x^2 + 4x + 4) + (4x^2 + 8x + 4) = 25$$
$$5x^2 + 12x + 8 = 25$$
$$5x^2 + 12x - 17 = 0$$
$$(5x+17)(x-1) = 0$$
$$x = -\frac{17}{5} \text{ or } x = 1$$

Substitute in the x values into the linear equation to get the corresponding y values

$$y = 2\left(-\frac{17}{5}\right) + 2 = -\frac{24}{5}$$
$$y = 2 \times 1 + 2 = 4$$

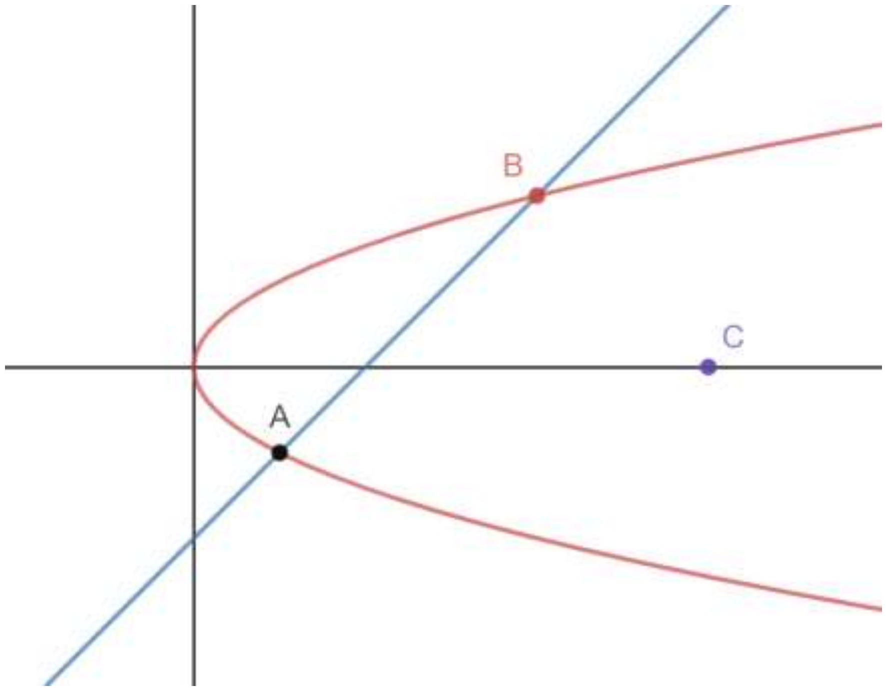
The co-ordinates of the intersections are: $\left(-\frac{17}{5}, -\frac{24}{5}\right)$ and $(1,4)$

Lines and Curves

The diagram shows the graphs of
$y^2 = x$ and $y = x - 2$

The graphs cross at the points A and B

The point C has co-ordinates (6,0)



- Without the use of a calculator, find the exact area of triangle ABC

Lines and Curves

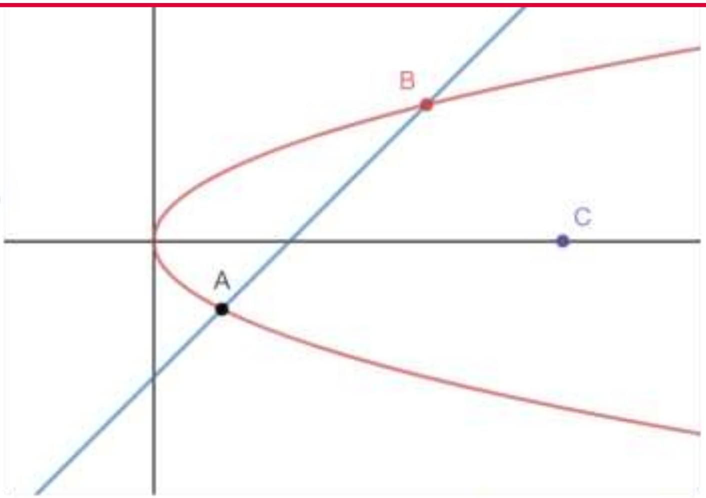
Solutions on the next slide….

Lines and Curves Solution

The diagram shows the graphs of
$y^2 = x$ and $y = x - 2$

The graphs cross at the points A and B

The point C has co-ordinates (6,0)



- Without the use of a calculator, find the exact area of triangle ABC

Solution:
Find the points A and B by substituting in $x - 2$ for y

$$(x-2)^2 = x$$
$$x^2 - 4x + 4 = x$$
$$x^2 - 5x + 4 = 0$$
$$(x-4)(x-1) = 0$$

So $x = 4$ or $x = 1$

Substitute these values back into $y = x - 2$

Gives $y = 4 - 2$ and $y = 1 - 2$

$y = 2$ and $y = -1$ so A is $(1, -1)$ and B is $(4,2)$

Solution continues on the next slide….

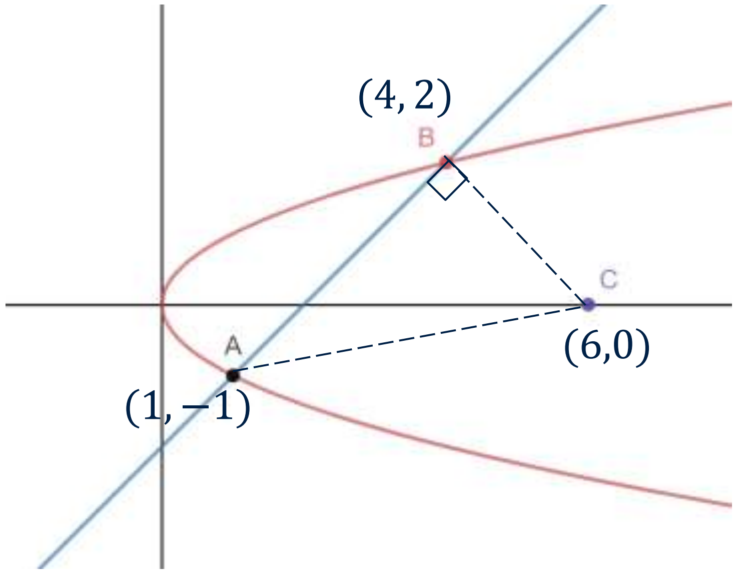
Lines and curves Solution

Having found the co-ordinates of A and B

We should now look at the gradients for AB and BC

Gradient of AB = $\frac{2-(-1)}{4-1} = \frac{3}{3} = 1$

Gradient of BC = $\frac{0-2}{6-4} = -\frac{2}{2} = -1$



As the gradients are negative reciprocals of each other this means that AB and BC are perpendicular and so triangle ABC is a right angled triangle.

To find the area of ABC we need to know the length BC and the height AB

$$AB^2 = (4-1)^2 + \left(2-(-1)\right)^2$$
$$AB = \sqrt{3^2 + 3^2}$$
$$AB = \sqrt{18} \ or \ 3\sqrt{2}$$

$$BC^2 = (6-4)^2 + (0-2)^2$$
$$BC = \sqrt{2^2 + (-2)^2}$$
$$BC = \sqrt{8} \ or \ 2\sqrt{2}$$

Area of triangle ABC is $\frac{1}{2} \times AB \times BC$ so $\frac{1}{2} \times 3\sqrt{2} \times 2\sqrt{2} = 3 \times \sqrt{2} \times \sqrt{2}$ which is 6 square units

Still want more?

Read about the history of Quadratic equations and how there are 101 uses for them!

Discover what is meant by a conic section and what on earth quadratics have to do with them.

Watch this video if you have ever been told that there are no solutions to a particular quadratic equation – because there are! They are not real though - welcome to imaginary maths! You can try a question for yourself here.

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