



**Advanced Mathematics
Support Programme®**

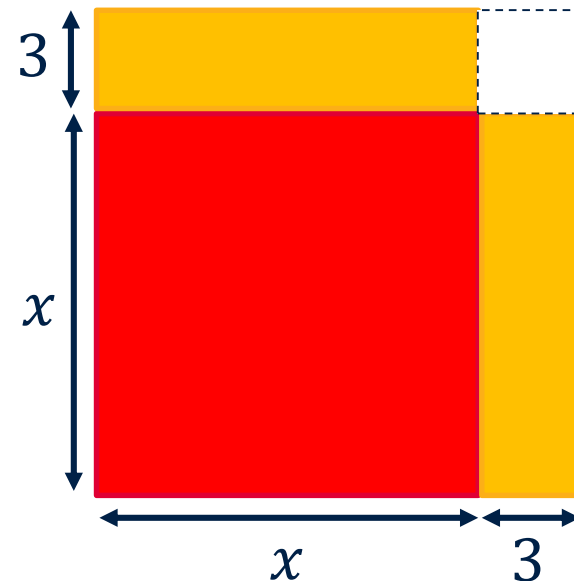
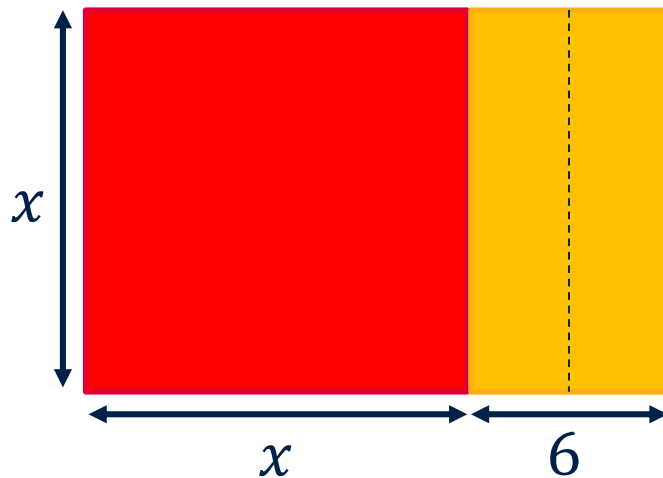
These are different forms of the same algebraic expression

$$x^2 + 6x = x(x + 6) = (x + 3)^2 - 9$$

expanded form

factorised form

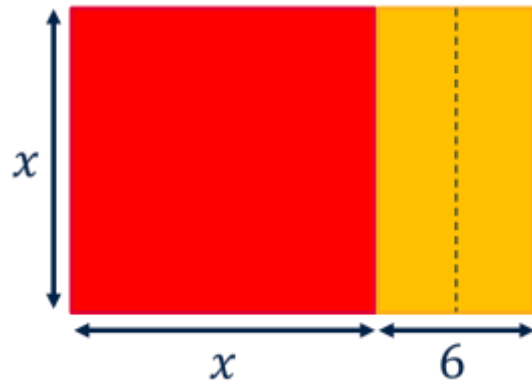
completed square form



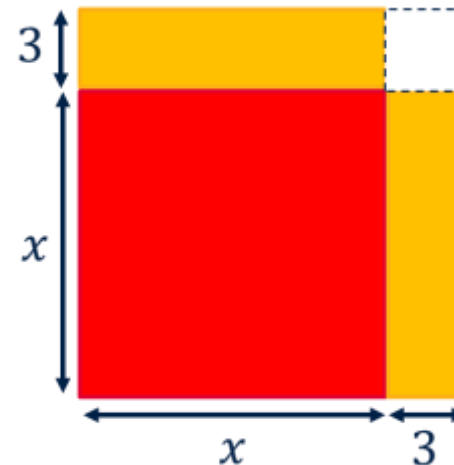
Do the diagrams help you see why this is called
Completing the square?

Think you've seen these diagrams before?

- They are very similar to the diagrams for the **Difference of Two Squares** – as seen previously in *Expanding Double Brackets*.



$$x^2 + 6x$$



Can you see this is the difference of two squares?

$$(x + 3)^2 - 3^2$$

$$((x + 3) - 3)((x + 3) + 3)$$

Collect like terms within the brackets

$$= x(x + 6)$$

$$= x^2 + 6x$$



Write these expressions in the form $(x + a)^2 + b$

1. $x^2 + 4x$

5. $x^2 - 12x + 41$

2. $x^2 + 4x + 5$

6. $k^2 + 10k - 2$

3. $y^2 - 8y$

7. $y^2 + 3y + 1$

4. $y^2 - 8y + 7$

8. $p^2 - 2p + 1$



Completing the square 1



Solutions on the next slide....



Write these expressions in the form $(x + a)^2 + b$

$$1. \quad x^2 + 4x \quad \rightarrow \quad = (x + 2)^2 - 4$$

$$2. \quad x^2 + 4x + 5 \quad \rightarrow \quad = (x + 2)^2 + 1$$

$$3. \quad y^2 - 8y \quad \rightarrow \quad = (y - 4)^2 - 16$$

$$4. \quad y^2 - 8y + 7 \quad \rightarrow \quad = (y - 4)^2 - 9$$



Write these expressions in the form $(x + a)^2 + b$

$$5. \quad x^2 - 12x + 41 \quad \longrightarrow \quad = (x - 6)^2 + 5$$

$$6. \quad k^2 + 10k - 2 \quad \longrightarrow \quad = (k + 5)^2 - 27$$

$$7. \quad y^2 + 3y + 1 \quad \longrightarrow \quad = \left(y + \frac{3}{2}\right)^2 - \frac{5}{4}$$

$$8. \quad p^2 - 2p + 1 \quad \longrightarrow \quad = (p - 1)^2$$



Write these expressions in the form $(x + a)^2 + b$

1. $x^2 + 10x$

5. $x^2 - 8x + 25$

2. $x^2 + 10x + 30$

6. $k^2 + 14k - 1$

3. $y^2 - 2y$

7. $y^2 + 5y + 6$

4. $y^2 - 2y + 3$

8. $t^2 + 6t + 9$



Completing the square 2



Solutions on the next slide....



Write these expressions in the form $(x + a)^2 + b$

$$1. \quad x^2 + 10x \quad \longrightarrow \quad = (x + 5)^2 - 25$$

$$2. \quad x^2 + 10x + 30 \quad \longrightarrow \quad = (x + 5)^2 + 5$$

$$3. \quad y^2 - 2y \quad \longrightarrow \quad = (y - 1)^2 - 1$$

$$4. \quad y^2 - 2y + 3 \quad \longrightarrow \quad = (y - 1)^2 + 2$$



Write these expressions in the form $(x + a)^2 + b$

$$5. \quad x^2 - 8x + 25 \quad \rightarrow \quad = (x - 4)^2 + 9$$

$$6. \quad k^2 + 14k - 1 \quad \rightarrow \quad = (k + 7)^2 - 50$$

$$7. \quad y^2 + 5y + 6 \quad \rightarrow \quad = \left(y + \frac{5}{2}\right)^2 - \frac{1}{4}$$

$$8. \quad t^2 + 6t + 9 \quad \rightarrow \quad = (t + 3)^2$$






It is important to be able to convert expressions between the different forms:

expanded form

factorised form

completed square form

In this problem there are 4 sets of three equivalent expressions, however, some expressions are missing. Match the sets and find the 3 missing expressions.




$a^2 - 2a - 8$		$a^2 - 8a + 15$
	$a^2 + 2a - 15$	$(a + 2)(a + 4)$
$(a + 1)^2 - 16$	$(a - 3)(a - 5)$	
$(a + 5)(a - 3)$	$(a - 1)^2 - 9$	$(a + 3)^2 - 1$

Different forms



Solutions on the next slide....



$a^2 - 2a - 8$	 $(a - 4)^2 - 1$	$a^2 - 8a + 15$
 $a^2 + 6a + 8$	$a^2 + 2a - 15$	$(a + 2)(a + 4)$
$(a + 1)^2 - 16$	$(a - 3)(a - 5)$	 $(a - 4)(a + 2)$
$(a + 5)(a - 3)$	$(a - 1)^2 - 9$	$(a + 3)^2 - 1$



What is the value of

$$\frac{\frac{(5^2 - 3^2)}{5 + 3} + \frac{(4^2 - 2^2)}{4 + 2} + \frac{(3^2 - 1^2)}{3 + 1}}{2} ?$$



Given that

$$55^2 - 45^2 = (55 + 45)(55 - 45) = 1000$$

and

$$60^2 - 40^2 = (60 + 40)(60 - 40) = 2000$$

Find numbers a and b such that $a^2 - b^2 = 3000$

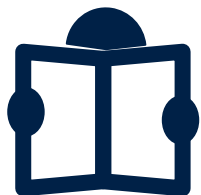
Find numbers c and d such that $c^2 - d^2 = 4000$

Find numbers e and f such that $e^2 - f^2 = 100\,000$

Extra Puzzles



Follow the [link](#) to the solutions



Learn about the history of solving quadratics and completing the square by learning about an Arab mathematician who is considered to be the founder of algebra.



Discover about removing cubes rather than squares. Does this activity help you consider the challenges involved in 'completing the cube'?



Watch this clip on parabolic flight. Think about the information you have learnt from completing the square and factorising, and how that links to the parabolic flight.

Contact the AMSP



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