Advanced Mathematics
Support Programme®

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Did you know?

To order fractions you can use the product of their diagonals

Compare these two fractions $\frac{5}{12}$ and $\frac{6}{13}$

5 x 13 = 65

6 x 12 = 72

$$\frac{\mathbf{5}}{\mathbf{12}} \quad \frac{\mathbf{6}}{\mathbf{13}}$$

as 72 > 65 then $\frac{6}{13}$ is larger than $\frac{5}{12}$

Compare these two fractions $\frac{42}{98}$ and $\frac{12}{28}$

42 x 28 = 1176

98 x 12 = 1176

$$\frac{\mathbf{42}}{\mathbf{98}} \quad \frac{\mathbf{12}}{\mathbf{28}}$$

This means $\frac{\mathbf{42}}{\mathbf{98}} = \frac{\mathbf{12}}{\mathbf{28}}$ so are **equivalent** fractions

If fractions are equivalent then the product of their diagonals will always be equal!

How could you use this to help you when rearranging expressions or equations with fractions ?

Rearranging Fractions

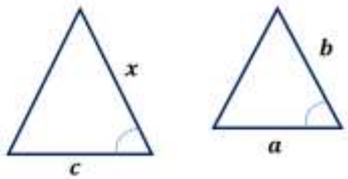
1. Rewrite the formula below to make time the subject.

$$\text{Speed} = \frac{\text{Distance}}{\text{time}}$$

2. Rearrange to make a the subject of $\frac{x}{y} = \frac{a}{b}$

3. Make x the subject of $tan\theta = \frac{y}{x}$

4. These triangles are similar.
Show that $x = \frac{cb}{a}$



5. Make a the subject of $x = \frac{h+k}{a}$

6. Make x the subject of $x + a = \frac{x+b}{c}$

7. Make a the subject of $\frac{1-a}{1+a} = \frac{x}{y}$

8. Make x the subject of $y(\sqrt{3} + \sqrt{2}\ )= x$
and write it in the form $x(\sqrt{a} + \sqrt{b}\ )$

Rearranging Fractions

Solutions on the next slide….

Rearranging Fractions Solutions

1. Rewrite the formula below to make time the subject.

$$\text{Speed} = \frac{\text{Distance}}{\text{time}}$$

$$Speed \times time = distance$$
$$time = \frac{distance}{Speed}$$

2. Rearrange to make a the subject of $\frac{x}{y} = \frac{a}{b}$

$$\frac{x}{y} = \frac{a}{b}$$
$$xb = ay$$
$$a = \frac{xb}{y}$$

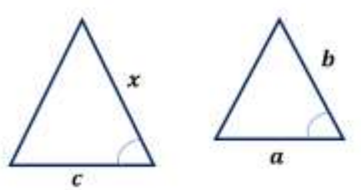
Remember the cross products of equivalent fractions are equal

3. Make x the subject of $tan\theta = \frac{y}{x}$

$$tan\theta = \frac{y}{x}$$
$$xtan\theta = y$$
$$x = \frac{y}{tan\theta}$$

4. These triangles are s

Show that $x = \frac{b}{a}$



Because the triangles are similar

$$\frac{x}{c} = \frac{b}{a}$$
$$ax = bc$$
$$x = \frac{bc}{a}$$ as required

Rearranging Fractions Solutions

5. Make a the subject of $x = \frac{h+k}{a}$

$$x = \frac{h+k}{a}$$
$$xa = h + k$$
$$a = \frac{h+k}{x}$$

6. Make x the subject of $x + a = \frac{x+b}{c}$

$$c(x+a) = x + b$$
$$cx + ca - x = b$$
$$cx - x = b - ca$$
$$x(c-1) = b - ca$$
$$x = \frac{b - ca}{c - 1}$$

7. Make a the subject of $\frac{1-a}{1+a} = \frac{x}{y}$

$$y(1-a) = x(1+a)$$
$$y - ay = x + xa$$
$$y - x = xa + ay$$
$$a(x+y) = y - x$$
$$a = \frac{y-x}{x+y}$$

8. Make x the subject of $y(\sqrt{3}+\sqrt{2}) = x$ and write it in the form $x(\sqrt{a}+\sqrt{b})$

$$y = \frac{x}{\sqrt{3}+\sqrt{2}}$$
$$y = \frac{x}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$
$$y = \frac{x\sqrt{3} - x\sqrt{2}}{3-2}$$
$$y = x(\sqrt{3}-\sqrt{2})$$

Rearranging Fractions 2

1. Make x the subject of

$$bc = \frac{x}{a}$$

2. Make e the subject of

$$x = \frac{y}{e^2}$$

3. Write a in terms of x, y, z and b

$$\frac{b - xa}{z} = y$$

4. Make v the subject of

$$C = \frac{v^2 - ta}{x}$$

5. Rearrange to make x the subject of

$$\frac{2}{x} + 5 = 6y$$

6. Make x the subject of

$$4F = F + \frac{a}{y + x}$$

7. Make y the subject of

$$\sqrt{\frac{m(y+a)}{y}} = g$$

8. A cylinder has a radius of $3cm$ and height, h. The total surface area = $30x\ cm^2$.

Find an expression for the surface area and write h in terms of x and π

Rearranging Fractions 2

Solutions on the next slide….

Rearranging Fractions 2 Solutions

1. Make x the subject of

$$bc = \frac{x}{a}$$

$$abc = x$$

2. Make e the subject of

$$x = \frac{y}{e^2}$$

$$e^2 x = y$$
$$e^2 = \frac{y}{x}$$
$$e = \sqrt{\frac{y}{x}}$$

3. Write a in terms of x, y, z and b

$$\frac{b - xa}{z} = y$$

$$b - xa = zy$$
$$-xa = zy - b$$
$$xa = b - zy$$
$$a = \frac{b - zy}{x}$$

Multiply by - 1

4. Make v the subject of

$$C = \frac{v^2 - ta}{x}$$

$$v^2 - ta = Cx$$
$$v^2 = Cx + ta$$
$$v = \pm\sqrt{Cx + ta}$$

Rearranging Fractions 2 Solutions

5. Rearrange to make x the subject of

$$\frac{2}{x} + 5 = 6y$$

$$\frac{2}{x} = 6y - 5$$
$$x(6y - 5) = 2$$
$$x = \frac{2}{6y - 5}$$

6. Make x the subject of

$$4F = F + \frac{a}{y + x}$$

$$3F = \frac{a}{y + x}$$
$$3Fy + 3Fx = a$$
$$3Fx = a - 3Fy$$
$$x = \frac{a - 3FY}{3F}$$

7. Make y the subject of

$$\sqrt{\frac{m(y+a)}{y}} = g$$

$$g^2 = \frac{my + ma}{y}$$
$$g^2 y = my + ma$$
$$g^2 y - my = ma$$
$$y(g^2 - m) = ma$$
$$y = \frac{ma}{g^2 - m}$$

8. A cylinder has a radius of $3cm$ and height, h. The total surface area = $30x\ cm^2$.

Find an expression for the surface area and write h in terms of x and π

Surface area of cylinder = $2\pi r^2 + 2\pi rh$

$$30x = (2\pi \times 3^2) + (2 \times 3 \times \pi \times h)$$
$$30x = 18\pi + 6\pi h$$
$$6\pi h = 30x - 18\pi$$
$$h = \frac{30x - 18\pi}{6\pi}$$
$$h = \frac{5x - 3\pi}{\pi}$$

Wrong Steps

Each expression has been rewritten in different ways.

- Which are not correct rearrangements?
- Give your reasons

	$c = \dfrac{3e^2}{d}$
A.	$d = 3e^2 - c$
B.	$cd = 3e^2$
C.	$\dfrac{d}{e^2} = \dfrac{c}{3}$
D.	$\dfrac{1}{3}c = \dfrac{e^2}{d}$
E.	$d = \dfrac{3e^2}{c}$

	$\dfrac{sinx}{4} = \dfrac{siny}{a}$
A.	$\dfrac{a}{4} = \dfrac{siny}{sinx}$
B.	$siny = \dfrac{4}{asinx}$
C.	$sinx = \dfrac{4siny}{a}$
D.	$asinx = 4siny$
E.	$a = \dfrac{sinx}{4siny}$

	$\dfrac{T-a}{T+a} = \dfrac{x}{y}$
A.	$x(T+a) = y(T-a)$
B.	$xy - ay = yT - ya$
C.	$a = \dfrac{y(T-a)}{x+y}$
D.	$xa + ya = yT - xT$
E.	$a = \dfrac{x+y}{yT-ya}$

Wrong Steps

Each expression has been rewritten in different ways.

- Which are not correct rearrangements?
- Give your reasons

	$a - \frac{b^2}{d} = ce$
A.	$b^2 = d(a + ce)$
B.	$a = ce + \frac{b^2}{d}$
C.	$\frac{b^2}{d} = a - ce$
D.	$\frac{b}{\sqrt{d}} = \sqrt{a} - \sqrt{ce}$
E.	$b = \pm\sqrt{d(a - ce)}$

	$y + b = \frac{ay + e}{b}$
A.	$by + b^2 = ay + e$
B.	$by - ay = e + b^2$
C.	$y = \frac{e - b^2}{b - a}$
D.	$e = b(y + b) - ay$
E.	$y(b - a) = \frac{e - b^2}{y}$

Wrong Steps

Solutions on the next slide….

Wrong Steps Solutions

Each expression has been rewritten in different ways.

- Which are not correct rearrangements?
- Give your reasons

	$c = \frac{3e^2}{d}$
A.	$d = 3e^2 - c$
B.	$cd = 3e^2$
C.	$\frac{d}{e^2} = \frac{c}{3}$
D.	$\frac{1}{3}c = \frac{e^2}{d}$
E.	$d = \frac{3e^2}{c}$

	$\frac{sinx}{4} = \frac{siny}{a}$
A.	$\frac{a}{4} = \frac{siny}{sinx}$
B.	$siny = \frac{4}{asinx}$
C.	$sinx = \frac{4siny}{a}$
D.	$asinx = 4siny$
E.	$a = \frac{sinx}{4siny}$

	$\frac{T-a}{T+a} = \frac{x}{y}$
A.	$x(T+a) = y(T-a)$
B.	$xy - ay = yT - ya$
C.	$a = \frac{y(T-a)}{x+y}$
D.	$xa + ya = yT - xT$
E.	$a = \frac{x+y}{yT-ya}$

Wrong Steps Solutions

Each expression has been rewritten in different ways.

- Which are not correct rearrangements?
- Give your reasons

$a-\frac{b^2}{d}=ce$	
A.	$b^2=d(a+ce)$
B.	$a=ce+\frac{b^2}{d}$
C.	$\frac{b^2}{d}=a\ -ce$
D.	$\frac{b}{\sqrt{d}}=\sqrt{a}-\sqrt{ce}$
E.	$b=\pm\sqrt{d(a-ce)}$

$y+b=\frac{ay+e}{b}$	
A.	$by+b^2=ay+e$
B.	$by-ay=e+b^2$
C.	$y=\frac{e-b^2}{b-a}$
D.	$e=b(y+b)-ay$
E.	$y(b-a)=\frac{e-b^2}{y}$

Prove It!

Using your rearranging skills can you prove each of the following

If $a = \frac{b}{b+c}$

Show that $\frac{a}{1-a} = \frac{b}{c}$

$\frac{n(n-1)}{2} + \frac{n(n+1)}{2}$ is a square number

$$\frac{2x+3}{4} - \frac{3x-2}{3} + \frac{1}{6} = \frac{19-6x}{12}$$

Prove It!

Solutions on the next slide….

Prove It Solutions

If $$a = \frac{b}{b+c}$$

Show that $$\frac{a}{a-1} = \frac{b}{c}$$

$\frac{a}{1} = \frac{b}{b+c}$	Make a into a fraction
$a(b+c) = b$	Using what we know about the product of the diagonals of equivalent fractions
$ab + ac = b$	Expand brackets
$ac = b - ab$	Make ac the subject
$ac = b(1-a)$	Factorise the right hand side
$b = \frac{ac}{1-a}$	Make b the subject
$\frac{b}{c} = \frac{a}{1-a}$	Divide both sides by c, expression as required

Prove It Solutions

$$\frac{n(n-1)}{2} + \frac{n(n+1)}{2}$$ is a square number

$$\frac{n^2 - n}{2} + \frac{n^2 + n}{2}$$

Expand brackets

$$\frac{n^2 - n + n^2 + n}{2}$$

Write as one fraction

$$\frac{2n^2}{2}$$

Simplify numerator

$$n^2$$

Cancel out factor of 2 so left with n^2 which is a square number as required

Prove It Solutions

$$\frac{2x+3}{4} - \frac{3x-2}{3} + \frac{1}{6} = \frac{19-6x}{12}$$

$$\frac{2x+3}{4} - \frac{3x-2}{3} + \frac{1}{6}$$

Concentrate on Left hand side

X 3 X 4 X 2

$$\frac{3(2x+3)}{12} - \frac{4(3x-2)}{12} + \frac{2}{12}$$

Make a common denominator

$$\frac{6x+9}{12} - \frac{12x-8}{12} + \frac{2}{12}$$

Expand brackets

$$\frac{6x+9-(12x-8)+2}{12}$$

Collect terms over single denominator

$$\frac{6x+9-12x+8+2}{12}$$

Simplify

$$\frac{19-6x}{12}$$

Left hand side is = to right hand side as required

Missing Steps

Complete the steps and fill in the blanks to find an expression for the area of triangle ABC

1. On the diagram draw a perpendicular line from A to BC
2. Label the perpendicular line h
3. Find an expression for the perpendicular height, h

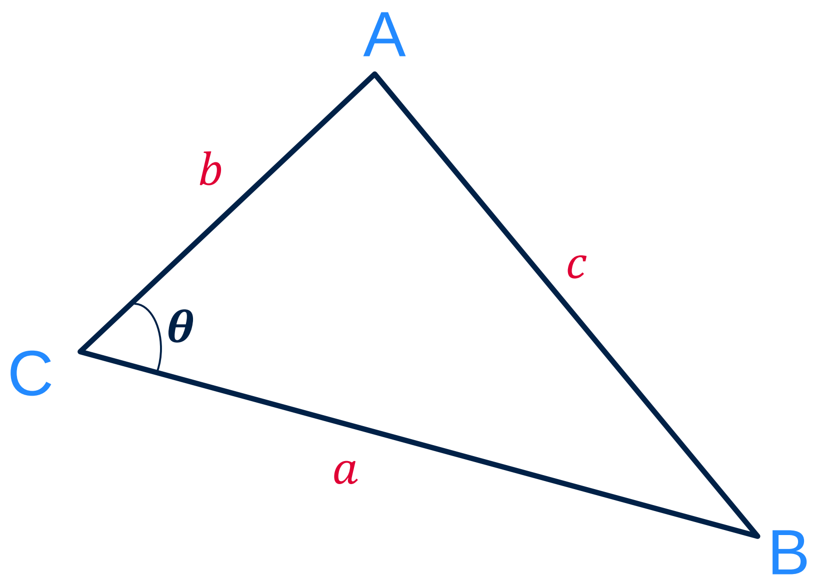
$h =$

4. Write down the expression for the base of the triangle

$base =$

5. Write down an expression to find the area of this triangle using your expressions for $base$ and $perpendicular\ height$

$area =$



Hint: You might want to introduce some trigonometry at step 3.

Missing Steps

Solutions on the next slide….

Missing Steps Solutions

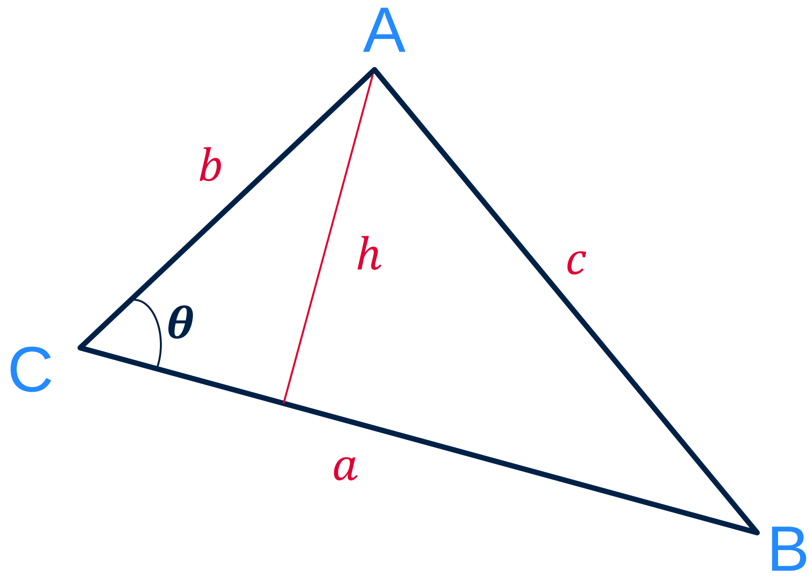
Complete the steps and fill in the blanks to find an expression for the area of triangle ABC

1. On the diagram draw a perpendicular line from A to BC

2. Label the perpendicular line h

3. Find an expression for the perpendicular height, h

$$Sin\theta = \frac{h}{b} \quad so \quad h = b\ sin\theta$$



4. Write down the expression for the base of the triangle

$$base = a$$

5. Write down an expression to find the area of this triangle using your expressions for $base$ and $perpendicular\ height$

$$Area = \frac{1}{2} base\ x\ perpendicular\ height$$

$$A = \frac{1}{2} a \times bsin\theta \quad \textbf{or} \quad A = \frac{1}{2} ab\ sin\theta$$

Still want more?

Read about how the rearrangement of algebraic expressions can be used in many real life contexts including proving the quadratic formulae!

Discover how trigonometry was developed to become the study of algebraic ratios from numeric beginnings by comparing the merkhet (not comparing the meerkat!)

Play with Lego, visit Paris and do maths all at the same time? It is possible through Helices!

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