Answers

to

Diagnostic

Tests

1. Evaluating Expressions

A) $pqr = (2)(-1)(3) = -6$

B) $\dfrac{3p-2q}{4r} = \dfrac{3(2)-2(-1)}{4(3)} = \dfrac{6+2}{12} = \dfrac{8}{12} = \dfrac{2}{3}$

C) $4 - 2p - q = 4 - 2(2) - (-1) = 4 - 4 + 1 = 1$

D) $3p^2 - 5q^2 = 3(2)^2 - 5(-1)^2 = 3(4) - 5(1) = 12 - 5 = 7$

E) $\sqrt{r^2 + 4p^2} = \sqrt{3^2 + 4(2)^2} = \sqrt{9 + 4(4)} = \sqrt{9+16} = \sqrt{25} = 5$

2. Simplifying Expressions 1

A) $4x - 2y + 3x + 8y - 2z = 7x + 6y - 2z$

B) $2a - 3a^2 + 5a + 5 + 7a^2 = 4a^2 + 7a + 5$

C) $5a \times 7a = 35a^2$

D) $\dfrac{4x^5}{2x^3} = 2x^2$

E) $2(a+3b) + 4(a-b) = 2a + 6b + 4a - 4b = 6a + 2b$

3. Removing Brackets (1 bracket)

A) $4(x+3) = 4x + 12$

B) $-3(x+2) = -3x - 6$

C) $3(4x + 7y) = 12x + 21y$

D) $2(3a - 5) = 6a - 10$

E) $-4(2x - 3) = -8x + 12$

F) $2a^2(3a - 2b) = 6a^3 - 4a^2b$

G) $-3a(5a + b - 2c) = -15a^2 - 3ab + 6ac$

4. Removing Brackets (2 brackets)

A) $(x+4)(x+3) = x^2 + 4x + 3x + 12 = x^2 + 7x + 12$

B) $(x+5)(x-1) = x^2 + 5x - x - 5 = x^2 + 4x - 5$

C) $(2x+3)(x-2) = 2x^2 + 3x - 4x - 6 = 2x^2 - x - 6$

D) $(2x-1)^2 = (2x-1)(2x-1) = 4x^2 - 2x - 2x + 1 = 4x^2 - 4x + 1$

5. Factorising (1 bracket)

A) $3x + 18 = 3(x + 6)$

B) $25x - 30y = 5(5x - 6y)$

C) $ab + 2b = b(a + 2)$

D) $5p + 15p^2 = 5p(1 + 3p)$

E) $cd^2 - 3cd^3 = cd^2(1 - 3d)$

6. Factorising (2 brackets)

A) $x^2 + 8x + 7 = (x+1)(x+7)$

B) $x^2 + 2x - 8 = (x+4)(x-2)$

C) $x^2 - 6x + 9 = (x-3)(x-3) = (x-3)^2$

D) $x^2 - 36 = (x+6)(x-6)$

E) $x^2 - 36y^2 = (x+6y)(x-6y)$

F) $3x^2 - 5x - 2 = (3x+1)(x-2)$

G) $12 + x - x^2 = (3+x)(4-x)$

H) $50 - 2x^2 = 2(25-x^2) = 2(5+x)(5-x)$

7. Linear Equations

A)
$$5x - 14 = 24$$
$$5x = 38$$
$$x = \frac{38}{5} = 7.6$$

B)
$$4x - 5 = 17 - 7x$$
$$4x + 7x = 17 + 5$$
$$11x = 22$$
$$x = 2$$

c)
$$6(2x-1) = 5(4x-6)$$
$$12x - 6 = 20x - 30$$
$$30 - 6 = 20x - 12x$$
$$24 = 8x$$
$$x = 3$$

D)
$$\frac{2x-5}{7} = 5$$
$$2x - 5 = 35$$
$$2x = 40$$
$$x = 20$$

E)
$$\frac{4}{x-2} = \frac{7}{2x-3}$$

$$4(2x-3) = 7(x-2)$$
$$8x - 12 = 7x - 14$$
$$8x - 7x = 12 - 14$$
$$x = -2$$

8. Quadratic Equations

A) $(x+4)(x+3) = 0$

$x = -4$ or $x = -3$

B) $(x+5)(x-1) = 0$

$x = -5$ or $x = 1$

c) $(2x+3)(x-2) = 0$

$x = -\frac{3}{2}$ or $x = 2$

D) $x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

$x = -3$ or $x = 1$

E) $x^2 - 4x = 0$

$x(x-4) = 0$

$x = 0$ or $x = 4$

F) $3x^2 - 5x - 2 = 0$

$(3x+1)(x-2) = 0$

$x = -\frac{1}{3}$ or $x = 2$

G) $x^2 - 4x - 11 = 0$

$x = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-11)}}{2(1)}$

$= \dfrac{4 \pm \sqrt{60}}{2}$

$x = -1.87$ (2dp) or $x = 5.87$ (2dp)

H) $3x^2 + 7x - 5 = 0$

$x = \dfrac{-7 \pm \sqrt{7^2 - 4(3)(-5)}}{2(3)}$

$= \dfrac{-7 \pm \sqrt{109}}{6}$

$x = 0.57$ (2dp) or

$x = -2.91$ (2dp)

9. Simultaneous Equations

A)

$$3x + 4y = 11 \qquad \text{①}$$
$$x + 7y = 15 \qquad \text{②}$$

| ① | $3x + 4y = 11$ | ③ |
| $3 \times$② | $3x + 21y = 45$ | ④ |

④ − ③ $\qquad 17y = 34$

$\qquad\qquad y = 2 \quad$: sub into ② $\qquad x + 14 = 15$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad x = 1$

$$\underline{x = 1 \;,\; y = 2}$$

B)

$$7x + 4y = 41 \qquad \text{①}$$
$$4x - 2y = 2 \qquad \text{②}$$

| | $7x + 4y = 41$ | ③ |
| $2 \times$② | $8x - 4y = 4$ | ④ |

③ + ④ $\qquad 15x = 45 \qquad x = 3$: sub into ② $\qquad 12 - 2y = 2$

$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad 2y = 10 \qquad y = 5$$

$$\underline{x = 3 \;,\; y = 5}$$

c)

$$x - 6y - 5 = 0 \qquad \text{①}$$
$$xy - 6 = 0 \qquad \text{②}$$

from ① $\qquad x = 6y + 5$

sub into ② $\qquad (6y + 5)y - 6 = 0$

$$\qquad\qquad\qquad 6y^2 + 5y - 6 = 0$$

$$\qquad\qquad\qquad (3y - 2)(2y + 3) = 0$$

$$\qquad\qquad\qquad y = \tfrac{2}{3} \Rightarrow x = 9$$

$$\qquad\qquad\qquad y = -\tfrac{3}{2} \Rightarrow x = -4$$

$$\underline{x = 9, \; y = \tfrac{2}{3}} \qquad \text{or} \qquad \underline{x = -4, \; y = \tfrac{-3}{2}}$$

10. Change the subject of the formula

A) $y = rx + p$

$y - p = rx$

$x = \dfrac{y-p}{r}$

F) $h = \sqrt{g - 2p}$

$h^2 = g - 2p$

$g = h^2 + 2p$

B) $h = abc$

$b = \dfrac{h}{ac}$

C) $f = \dfrac{g}{h}$

$fh = g$

$h = \dfrac{g}{f}$

G) $y = \dfrac{3x + 2}{2x - 1}$

$y(2x - 1) = 3x + 2$

$2xy - y = 3x + 2$

$2xy - 3x = 2 + y$

$x(2y - 3) = 2 + y$

$x = \dfrac{2 + y}{2y - 3}$

D) $y = \dfrac{s(ax - b)}{t}$

$yt = sax - sb$

$sax = yt + sb$

$x = \dfrac{yt + sb}{sa}$

E) $T = \dfrac{3W}{P - Q}$

$T(P - Q) = 3W$

$TP - TQ = 3W$

$TQ = TP - 3W$

$Q = \dfrac{TP - 3W}{T}$

or $P - Q = \dfrac{3W}{T}$ Check that these
are equivalent

$Q = P - \dfrac{3W}{T}$ ✓

Answers
to
Exercises

Answers to exercises

Exercise 1

1 a) $3(2) + 2 = 8$
b) $-1 - 3 = -4$
c) $5(-1) = -5$
d) $8(4) - 3 = 29$
e) $2(2)(-1)(4) = -16$
f) $3(-1) + 2 = -1$
g) $\frac{4}{4} = 1$

h) $2 + 3(-1) - 4 = -5$
i) $8(2) - 2(-1) + 5(4) = 38$
j) $2 - (-1) - 4 = -1$
k) $2(-1) - 4 - 2(2) = -10$
l) $\frac{3(2)(4)}{(-1)} = -24$

2 a) $2(4)(3) - (-2) = 26$
b) $\frac{4}{2} - 3 + \frac{(-2)}{2} = -2$

c) $\frac{5(4)}{3 - (-2)} = 4$

d) $\frac{3(4)(3)}{4 - (-2)} = 6$

e) $\frac{4(-2) + 3(3)}{-2 + 3} = 1$

f) $\frac{(3)(-2) - 4}{3 - (-2)} = -2$

3. a) $(2)^2 + (-3)^2 + (-1)^2$
$= 4 + 9 + 1$
$= 14$

b) $2(2)^2 - (-3)^2 + 2(-1)^2$
$= 8 - 9 + 2$
$= 1$

c) $\sqrt{4(2)^2 + (-3)^2}$
$= \sqrt{16 + 9}$
$= \sqrt{25}$
$= 5$

d) $[2(-3)]^2 - 2(-3)^2$
$= (-6)^2 - 2(9)$
$= 36 - 18$
$= 18$

e) $\sqrt[3]{3(2^3 - (-1)^3)}$
$= \sqrt[3]{3(8 - (-1))}$
$= \sqrt[3]{3(8 + 1)}$
$= \sqrt[3]{3(9)}$
$= \sqrt[3]{27}$
$= 3$

Exercise 2 A

1. $5x + 2x - 7x = 0$
2. $5y - 2y - 9y = -6y$
3. $2q - 7p + 4p + 5q = 7q - 3p$
4. $8a - 2b + 7a + 6b = 15a + 4b$
5. $3ab - 2ab + 9ab = 10ab$
6. $t \times 5t = 5t^2$
7. $h \times 4h \times 3h = 12h^3$
8. $w \times w \times 3w = 3w^3$
9. $4d \div 2d = 2$
10. $20p^3 \div 5p^2 = 4p$
11. $p^3 \times p^5 = p^8$
12. $35d^5 \div 7d^4 = 5d$
13. $6c \times 5c = 30c^2$
14. $6y \times 8y^2 = 48y^3$
15. $(9t)^2 = 81t^2$
16. $(3r)^3 = 27r^3$
17. $\sqrt{49t^2} = 7t$
18. $\sqrt[3]{27b^6} = 3b^2$
19. $qp + 2qp - r = 3qp - r$
20. $h + 4h^2 - 2h + 7h^2 = 11h^2 - h$
21. $pq + qp = 2pq$
22. $a^3 - 5a^3 - 7a^3 = -11a^3$
23. $3(2a + 3b) + 2(a - 5b) = 6a + 9b + 2a - 10b = 8a - b$
24. $7(x^2 - 2x + 21) = 7x^2 - 14x + 147$
25. $5(d - 2) - (d - 5) = 5d - 10 - d + 5 = 4d - 5$
26. $4(w - 3) + 7(2w - 3) = 4w - 12 + 14w - 21 = 18w - 33$
27. $8pq \div 4qr = \dfrac{8pq}{4qr} = \dfrac{2p}{r}$

28. $(2ab^2)^2 = 4a^2b^4$
29. $(3xy)^3 = 27x^3y^3$
30. $(ab)^2 \times (ab^2)^3 = a^2b^2 \times a^3b^6 = a^5b^8$

Exercise 2B

1. $p \times -q = -pq$
2. $(-4a) \div (-2a) = 2$
3. $p \times (-p) = -p^2$
4. $4t \times (-5t) = -20t^2$
5. $27y^3 \div (-3y) = -9y^2$
6. $4a^2b^3 \times 2ab^2 = 8a^3b^5$
7. $(-7xy) \times (-3x^2y) = 21x^3y^2$
8. $p \times 3p = 3p^2$
9. $(-6p) \times (3p) = -18p^2$
10. $\dfrac{t}{2} \times \dfrac{t}{3} = \dfrac{t^2}{6}$

11. $\dfrac{t}{2} \div \dfrac{t}{3} = \dfrac{t}{2} \times \dfrac{3}{t} = \dfrac{3}{2}$

12. $\dfrac{t^3}{4} \times \dfrac{t^2}{3} = \dfrac{t^5}{12}$

13. $\dfrac{t^3}{4} \div \dfrac{t^2}{3} = \dfrac{t^3}{4} \times \dfrac{3}{t^2} = \dfrac{3t}{4}$

14. $2ab^2 \times (-3b^2) = -6ab^4$
15. $(w - 2p) - (3p - 4w) = w - 2p - 3p + 4w = 5w - 5p$
16. $(-r) \times (-2r) \times (-r) = -2r^3$
17. $y^2 \times (-2y) \times (-3y) = 6y^4$
18. $-3(x - 2y) - 4(3y - 5x) = -3x + 6y - 12y + 20x = 17x - 6y$
19. $(4z + 8y) \times \dfrac{y}{2} = (2z + 4y)y = 2zy + 4y^2$

20. $(4z + 8y) \div \dfrac{y}{2} = (4z + 8y) \times \dfrac{2}{y} = \dfrac{8z}{y} + 16$

1. $6(x+7) = 6x + 42$

2. $3(x-5) = 3x - 15$

3. $-2(a+2b) = -2a - 4b$

4. $4(3x - 2y) = 12x - 8y$

5. $-(y-3x) = -y + 3x$

6. $8x(x+5) = 8x^2 + 40x$

7. $p(3 - 2p) = 3p - 2p^2$

8. $3w(4x+5w) = 12wx + 15w^2$

9. $3k(3k+1) = 9k^2 + 3k$

10. $-9a^2(a-3) = -9a^3 + 27a^2$

11. $a(a + 2b - c) = a^2 + 2ab - ac$

12. $p(3p - 2q - 4r) = 3p^2 - 2pq - 4pr$

13. $8(x - 4y) = 8x - 32y$

14. $-3x(x^2 - 2) = -3x^3 + 6x$

15. $3a(a^2 - b^2) = 3a^3 - 3ab^2$

16. $y(x - y - 3) = yx - y^2 - 3y$

17. $2x(4x + 5y - 3z + 5) = 8x^2 + 10xy - 6xz + 10x$

18. $-5c^2(3 - 4c) = -15c^2 + 20c^3$

19. $9y(1 - y^2) = 9y - 9y^3$

20. $-5d(2 - 3d) = -10d + 15d^2$

Exercise 4

1. $(x+3)(x+2) = x^2 + 3x + 2x + 6 = x^2 + 5x + 6$

2. $(x-5)(x+4) = x^2 - 5x + 4x - 20 = x^2 - x - 20$

3. $(x+3)(x-5) = x^2 + 3x - 5x - 15 = x^2 - 2x - 15$

4. $(x-2)(x-7) = x^2 - 2x - 7x + 14 = x^2 - 9x + 14$

5. $(2x+1)(x+3) = 2x^2 + x + 6x + 3 = 2x^2 + 7x + 3$

6. $(a+2b)(a-b) = a^2 + 2ab - ab - 2b^2 = a^2 + ab - 2b^2$

7. $(3x-4)(2x-1) = 6x^2 - 8x - 3x + 4 = 6x^2 - 11x + 4$

8. $(x+3)^2 = (x+3)(x+3) = x^2 + 6x + 9$

9. $(3k+1)^2 = (3k+1)(3k+1) = 9k^2 + 6k + 1$

10. $(a-3)^2 = (a-3)(a-3) = a^2 - 6a + 9$

11. $(x-y)(x+2y) = x^2 - xy + 2xy - 2y^2 = x^2 + xy - 2y^2$

12. $(3-2p)(2-3p) = 6 - 4p - 9p + 6p^2 = 6 - 13p + 6p^2$

13. $(x-4y)(y-2x) = xy - 4y^2 - 2x^2 + 8xy = 9xy - 4y^2 - 2x^2$

14. $(x^2-2)(x+1) = x^3 - 2x + x^2 - 2$

15. $(a-b)(a+b) = a^2 - ab + ab - b^2 = a^2 - b^2$

16. $(a+b)(a+b) = a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$

17. $(3a+1)(3a-1) = 9a^2 + 3a - 3a - 1 = 9a^2 - 1$

18. $(3a+1)(3a+1) = 9a^2 + 3a + 3a + 1 = 9a^2 + 6a + 1$

19. $(y+2)(1-y^2) = y + 2 - y^3 - 2y^2$

20. $(4d+3)(2-3d) = 8d + 6 - 12d^2 - 9d = 6 - d - 12d^2$

Exercise 5

1. $3x + 9 = 3(x + 3)$
2. $4x - 4 = 4(x - 1)$
3. $49x - 21y = 7(7x - 3y)$
4. $pq - pr = p(q - r)$
5. $bx^2 + bx = bx(x + 1)$
6. $3x^2 - 2x = x(3x - 2)$
7. $ab^2 + 4a^2b = ab(b + 4a)$
8. $2\pi r^2 + \pi r h = \pi r(2r + h)$
9. $45a^2 - 63ab^2 = 9a(5a - 7b^2)$
10. $x^3 + 2x^2 = x^2(x + 2)$
11. $hc^2 - 3h^3c = hc(c - 3h^2)$
12. $pq - 5qp^2 = pq(1 - 5p)$
13. $4ab^2c^3 - 6a^2bc^3 = 2abc^3(2b - 3a)$
14. $\frac{x}{2} + \frac{y}{4} + \frac{z}{8} = \frac{1}{8}(4x + 2y + z)$

15. $18c^2 - 6c + 36c^3 = 6c(3c - 1 + 6c^2)$
16. $5x^2 - 30x = 5x(x - 6)$
17. $7y^4 + 3y^3 = y^3(7y + 3)$
18. $5t^2r^3 + 10t^2r^4 = 5t^2r^3(1 + 2r)$
19. $\sqrt{x} + 5y\sqrt{x} = \sqrt{x}(1 + 5y)$
20. $\sqrt{x} + 4x = \sqrt{x}(1 + 4\sqrt{x})$

$$\underline{\text{Exercise } 6}$$

1. $x^2 + 6x + 8 = (x+2)(x+4)$

2. $x^2 - 3x + 2 = (x-1)(x-2)$

3. $x^2 + 2x - 15 = (x+5)(x-3)$

4. $x^2 - 5x - 14 = (x+2)(x-7)$

5. $x^2 + 9x + 20 = (x+5)(x+4)$

6. $2x^2 - 9x - 5 = (2x+1)(x-5)$

7. $2x^2 + 13x + 15 = (2x+3)(x+5)$

8. $2x^2 - 5x + 3 = (2x - 3)(x - 1)$

9. $x^2 - 49 = (x+7)(x-7)$

10. $15q^2 - 26q - 21 = (5q + 3)(3q - 7)$

11. $35 - x - 6x^2 = (5 + 2x)(7 - 3x)$

12. $28 - 3x - x^2 = (4 - x)(7 + x)$

13. $x^2 - 18x + 81 = (x - 9)(x - 9) = (x-9)^2$

14. $16a^2 - 25b^2 = (4a + 5b)(4a - 5b)$

15. $p^2 - 2pq + q^2 = (p - q)(p - q) = (p-q)^2$

16. $100 - 36x^2 = (10 - 6x)(10 + 6x)$
                    $= 4(5 - 3x)(5 + 3x)$

Exercise 7A

1. $x + 7 = 19$

    $x = 19 - 7$

    $x = 12$

2. $3p - 8 = 13$

    $3p = 13 + 8$

    $3p = 21$

    $p = 7$

3. $7x = 42$

    $x = 6$

4. $14 - 3x = 8$

    $3x = 14 - 8$

    $3x = 6$

    $x = 2$

5. $9x - 3x + 5x = 33$

    $11x = 33$

    $x = 3$

6. $6t - 7 = 23$

    $6t = 23 + 7$

    $6t = 30$

    $t = 5$

7. $3(x - 7) = 12$

    $x - 7 = 4$

    $x = 11$

8. $2k + 5 = 7k - 15$

    $7k - 2k = 5 + 15$

    $5k = 20$

    $k = 4$

9. $5(9 - 2w) = 3w$

    $45 - 10w = 3w$

    $45 = 13w$

    $w = \frac{45}{13}$

10. $5(d + 2) - 3(d - 5) = 29$

    $5d + 10 - 3d + 15 = 29$

    $2d = 29 - 25$

    $2d = 4$

    $d = 2$

11. $0.6x = 5.4$

    $x = \frac{5.4}{0.6} = 9$

12. $2.7x - 4.1 = 8.4 - 2.3x$

    $2.7x + 2.3x = 8.4 + 4.1$

    $5x = 12.5$

    $x = \frac{12.5}{5} = 2.5$

13. $\frac{5x - 7}{3} = 8$

    $5x - 7 = 24$

    $5x = 31$

    $x = \frac{31}{5} = 6.2$

14. $5 - 9f = 3f + 29$

    $3f + 9f = 5 - 29$

    $12f = -24$

    $f = -2$

15. $8(7 - 2x) = 4x + 3$

    $56 - 16x = 4x + 3$

    $4x + 16x = 56 - 3$

    $20x = 53$

    $x = 2.65$

16. $$\frac{3}{x-2} = \frac{4}{x+4}$$

$$3(x+4) = 4(x-2)$$
$$3x + 12 = 4x - 8$$
$$4x - 3x = 12 + 8$$
$$x = 20$$

Exercise 7B

1. $\dfrac{5}{x-5} = \dfrac{8}{2x-1}$

$5(2x-1) = 8(x-5)$

$10x - 5 = 8x - 40$

$10x - 8x = -40 + 5$

$2x = -35$

$x = \dfrac{-35}{2} = -17.5$

2. $\dfrac{x+3}{4} - \dfrac{x-3}{5} = 2$

$\dfrac{5(x+3) - 4(x-3)}{20} = 2$

$5x + 15 - 4x + 12 = 40$

$x = 40 - 27$

$x = 13$

3. $\dfrac{x-2}{x-3} = 5$

$x - 2 = 5(x-3)$

$x - 2 = 5x - 15$

$5x - x = -2 + 15$

$4x = 13$

$x = \dfrac{13}{4} = 3.25$

4. $\dfrac{3q-5}{4} - \dfrac{9-2q}{3} = 0$

$\dfrac{3(3q-5) - 4(9-2q)}{12} = 0$

$9q - 15 - 36 + 8q = 0$

$17q = 51$

$q = 3$

5. $\dfrac{3w-5}{2} = \dfrac{6-3w}{3}$

$3(3w-5) = 2(6-3w)$

$9w - 15 = 12 - 6w$

$15w = 27$

$w = \dfrac{27}{15} = 1.8$

6. $\dfrac{2-t}{5} = \dfrac{t}{3}$

$3(2-t) = 5t$

$6 - 3t = 5t$

$6 = 8t$

$t = \dfrac{6}{8} = \dfrac{3}{4} = 0.75$

7. $\dfrac{3x-1}{2} - \dfrac{2x-1}{3} = \dfrac{1-2x}{4}$

$\dfrac{3(3x-1) - 2(2x-1)}{6} = \dfrac{1-2x}{4}$

$\dfrac{5x - 1}{6} = \dfrac{1-2x}{4}$

$4(5x-1) = 6(1-2x)$

$20x - 4 = 6 - 12x$

$32x = 10$

$x = \dfrac{10}{32} = \dfrac{5}{16}$

8. $\dfrac{3}{2(x-4)} = \dfrac{5}{3(x-2)}$

$9(x-2) = 10(x-4)$

$9x - 18 = 10x - 40$

$10x - 9x = -18 + 40$

$x = 22$

Exercise 8A

1. $x^2 + 4x - 32 = 0$
   $(x + 8)(x - 4) = 0$
   $x = -8$ or $x = 4$

2. $x^2 + 9x + 20 = 0$
   $(x + 5)(x + 4) = 0$
   $x = -5$ or $x = -4$

3. $x^2 - 25 = 0$
   $(x + 5)(x - 5) = 0$
   $x = -5$ or $x = 5$

4. $7x^2 - 63 = 0$
   $7(x^2 - 9) = 0$
   $7(x + 3)(x - 3) = 0$
   $x = -3$ or $x = 3$

5. $(x - 2)(x - 7) = 0$
   $x = 2$ or $x = 7$

6. $x(x - 6) = 0$
   $x = 0$ or $x = 6$

7. $2x(5 - x) = 0$
   $x = 0$ or $x = 5$

8. $(2x + 1)(x + 3) = 0$
   $x = -\frac{1}{2}$ or $x = -3$

9. $(2 - x)(3 + 2x) = 0$
   $x = 2$ or $x = -\frac{3}{2}$

10. $x^2 + x - 72 = 0$
    $(x + 9)(x - 8) = 0$
    $x = -9$ or $x = 8$

11. $x^2 + 5x = 0$
    $x(x + 5) = 0$
    $x = 0$ or $x = -5$

12. $14y^2 = 29y - 12$
    $14y^2 - 29y + 12 = 0$
    $(2y - 3)(7y - 4) = 0$
    $y = \frac{3}{2}$ or $y = \frac{4}{7}$

13. $4x^2 - 4x - 3 = 0$
    $(2x - 3)(2x + 1) = 0$
    $x = \frac{3}{2}$ or $x = -\frac{1}{2}$

14. $y^2 = 10y - 25$
    $y^2 - 10y + 25 = 0$
    $(y - 5)^2 = 0$
    $y = 5$

15. $9x + 28 = 9x^2$
    $9x^2 - 9x - 28 = 0$
    $(3x - 7)(3x + 4) = 0$
    $x = \frac{7}{3}$ or $x = -\frac{4}{3}$

16. $5x^2 - 80 = 0$
    $5(x^2 - 16) = 0$
    $5(x + 4)(x - 4) = 0$
    $x = -4$ or $x = 4$

17. $x^2 - 8x - 33 = 0$
    $(x + 3)(x - 11) = 0$
    $x = -3$ or $x = 11$

18. $4 - x - 5x^2 = 0$
    $5x^2 + x - 4 = 0$
    $(5x - 4)(x + 1) = 0$
    $x = \frac{4}{5}$ or $x = -1$

19. $p(5 - p) = 0$
    $p = 0$ or $p = 5$

20. $t(4 + 7t) = 0$
    $t = 0$ or $t = -\frac{4}{7}$

Exercise 8B

1. $x^2 - 3x - 1 = 0$

$x = \dfrac{3 \pm \sqrt{9 + 4}}{2} = \dfrac{3 \pm \sqrt{13}}{2}$

$x = 3.31$ or $x = -0.30$ (2dp)

2. $3x^2 - 4x - 1 = 0$

$x = \dfrac{4 \pm \sqrt{16 + 12}}{6} = \dfrac{4 \pm \sqrt{28}}{6}$

$x = 1.55$ or $x = -0.22$ (2dp)

3. $5x^2 - 3x - 5 = 0$

$x = \dfrac{3 \pm \sqrt{9 + 100}}{10} = \dfrac{3 \pm \sqrt{109}}{10}$

$x = 1.34$ or $x = -0.74$ (2dp)

4. $x(x - 5) + 3x(x + 3) = 7$

$x^2 - 5x + 3x^2 + 9x = 7$

$4x^2 + 4x - 7 = 0$

$x = \dfrac{-4 \pm \sqrt{16 + 112}}{8} = \dfrac{-4 \pm \sqrt{128}}{8}$

$x = 0.91$ or $x = -1.91$ (2dp)

5. $x(x - 7) = 13$

$x^2 - 7x - 13 = 0$

$x = \dfrac{7 \pm \sqrt{49 + 52}}{2} = \dfrac{7 \pm \sqrt{101}}{2}$

$x = 8.52$ or $x = -1.52$

6. $x^2 + 5x = 11$

$x^2 + 5x - 11 = 0$

$x = \dfrac{-5 \pm \sqrt{25 + 44}}{2} = \dfrac{-5 \pm \sqrt{69}}{2}$

$x = 1.65$ or $x = -6.65$

7. $(2x-3)^2 = 8$

$4x^2 - 12x + 9 = 8$

$4x^2 - 12x + 1 = 0$

$x = \dfrac{12 \pm \sqrt{144 - 16}}{8} = \dfrac{12 \pm \sqrt{128}}{8}$

$x = 2.91$ or $x = 0.09$ (2dp)

8. $4x^2 + 2x - 3 = 0$

$x = \dfrac{-2 \pm \sqrt{4 + 48}}{8} = \dfrac{-2 \pm \sqrt{52}}{8}$

$x = 0.65$ or $x = -1.15$ (2dp)

9. $y^2 + 5y - 3 = 0$

$y = \dfrac{-5 \pm \sqrt{25 + 12}}{2} = \dfrac{-5 \pm \sqrt{37}}{2}$

$y = 0.54$ or $y = -5.54$ (2dp)

10. $\dfrac{5}{x-3} - \dfrac{1}{x} = 3$

$\dfrac{5x - (x-3)}{x(x-3)} = 3$

$5x - x + 3 = 3x(x-3)$

$4x + 3 = 3x^2 - 9x$

$3x^2 - 13x - 3 = 0$

$x = \dfrac{13 \pm \sqrt{169 + 36}}{6} = \dfrac{13 \pm \sqrt{205}}{6}$

$x = 4.55$ or $x = -0.22$ (2dp)

Exercise 9A

1. $x - y = 6$ ①
   $x + 3y = 7$ ②

   $3 \times ①$ $3x - 3y = 18$
   ② $x + 3y = 7$
         $4x = 25$
         $x = \frac{25}{4}$ $y = \frac{1}{4}$

2. $x + 3y = 11$ ①
   $5x - 2y = 4$ ②

   $5 \times ①$ $5x + 15y = 55$
   ② $5x - 2y = 4$
   (subtract)
         $17y = 51$
         $y = 3$ $x = 2$

3. $3x + 2y = 13$ ①
   $8x - 7y = 10$ ②

   $7 \times ①$ $21x + 14y = 91$
   $2 \times ②$ $16x - 14y = 20$
         $37x = 111$
         $x = 3$ $y = 2$

4. $8x - 12y + 5 = 0$ ①
   $28x - 20y + 1 = 0$ ②

   $12 \times ②$ $336x - 240y + 12 = 0$
   $20 \times ①$ $160x - 240y + 100 = 0$
   subtract $176x - 88 = 0$
         $x = \frac{1}{2}$ $y = \frac{3}{4}$

5.

$$2x - 2y = 12 \quad \text{①}$$
$$x + 5y = 0 \quad \text{②}$$
$$x = -5y \quad \text{sub into ①}$$

$$2(-5y) - 2y = 12$$
$$-10y - 2y = 12$$
$$-12y = 12$$
$$\underline{y = -1} \qquad \underline{x = 5}$$

6.

$$3(2-x) + 2(3+y) = 11 \quad \text{①}$$
$$4(x-2) + 3(y-1) = 13 \quad \text{②}$$

① becomes
$$6 - 3x + 6 + 2y = 11$$
$$\text{ie} \quad 3x - 2y = 1 \quad \text{③}$$

② becomes
$$4x - 8 + 3y - 3 = 13$$
$$\text{ie} \quad 4x + 3y = 24 \quad \text{④}$$

$3 \times$ ③
$$9x - 6y = 3$$
$2 \times$ ④
$$8x + 6y = 48$$

$$17x = 51$$

$$\underline{x = 3} \qquad \underline{y = 4}$$

7.
$$x - 8y + 20 = 0 \quad \textcircled{1}$$
$$9x + y = 39 \quad \textcircled{2}$$

from $\textcircled{2}$ $y = 39 - 9x$
sub into $\textcircled{1}$
$$x - 8(39 - 9x) + 20 = 0$$
$$x - 312 + 72x + 20 = 0$$
$$73x = 292$$
$$\underline{x = 4} \qquad \underline{y = 3}$$

8.
$$x + y = 3 \quad \textcircled{1}$$
$$3x + 2y = 7 \quad \textcircled{2}$$
$2 \times \textcircled{1}$ $\quad 2x + 2y = 6$

$$\underline{x = 1} \qquad \underline{y = 2}$$

Exercise 9B

1.
$$x + 2y = 5 \quad \textcircled{1}$$
$$xy = 2 \quad \textcircled{2}$$
from $\textcircled{1}$ $x = 5 - 2y$
sub into $\textcircled{2}$ $(5 - 2y)y = 2$

$$5y - 2y^2 = 2$$
$$2y^2 - 5y + 2 = 0$$
$$(2y - 1)(y - 2) = 0$$
$$y = \tfrac{1}{2} \quad \text{or} \quad y = 2$$
$$\downarrow \qquad\qquad \downarrow$$
$$x = 4 \qquad\qquad x = 1$$

ie
$$\underline{x = 1, \ y = 2} \qquad \text{or} \qquad \underline{x = 4, \ y = \tfrac{1}{2}}$$

2.
$$x - y = 2 \quad \text{①}$$
$$x^2 + 3y = 4 \quad \text{②}$$

from ① $y = x - 2$

sub into ② $x^2 + 3(x - 2) = 4$

$$x^2 + 3x - 6 - 4 = 0$$
$$x^2 + 3x - 10 = 0$$
$$(x + 5)(x - 2) = 0$$
$$x = -5 \quad \text{or} \quad x = 2$$
$$\downarrow \qquad\qquad\qquad \downarrow$$
$$y = -7 \qquad\qquad y = 0$$

ie

$$\underline{x = 2, \ y = 0} \qquad \text{or} \qquad \underline{x = -5, \ y = -7}$$

3.
$$y = 3x - 8 \quad \text{①}$$
$$2x^2 - y^2 = 4 \quad \text{②}$$

sub ① into ② $2x^2 - (3x - 8)^2 = 4$

$$2x^2 - (9x^2 - 48x + 64) = 4$$
$$7x^2 - 48x + 68 = 0$$
$$(7x - 34)(x - 2) = 0$$
$$x = \frac{34}{7} \qquad\qquad x = 2$$
$$\downarrow \qquad\qquad\qquad \downarrow$$
$$y = \frac{46}{7} \qquad\qquad y = -2$$

ie

$$\underline{x = \frac{34}{7}, \ y = \frac{46}{7}} \qquad \text{or} \qquad \underline{x = 2, \ y = -2}$$

4. $x + 2y + 5 = 0$ ①

$x^2 - x + y = 0$ ②

from ② $y = x - x^2$

sub into ①

$$x + 2(x - x^2) + 5 = 0$$

$$2x^2 - 3x - 5 = 0$$

$$(2x - 5)(x + 1) = 0$$

$x = \dfrac{5}{2}$ $x = -1$

↓   ↓

$y = -\dfrac{15}{4}$ $y = -2$

ie $x = \dfrac{5}{2}, \ x = \dfrac{-15}{4}$ or $x = -1, \ y = -2$

5. $x + y = 6$ ①

$(x - 2)^2 + y = 10$ ②

from ① $y = 6 - x$

$$(x - 2)^2 + 6 - x = 10$$

$$x^2 - 4x + 4 + 6 - x = 10$$

$$x^2 - 5x = 0$$

$$x(x - 5) = 0$$

$x = 0$ $x = 5$

↓   ↓

$y = 6$ $y = 1$

ie $x = 0, \ y = 6$ or $x = 5, \ y = 1$

6.
$$2y + x = 10 \qquad ①$$
$$2x^2 + 3y^2 = 7xy \qquad ②$$

from ① $\qquad x = 10 - 2y$

sub into ②

$$2(10-2y)^2 + 3y^2 = 7(10-2y)y$$

$$2(100 - 40y + 4y^2) + 3y^2 = 70y - 14y^2$$
$$200 - 80y + 8y^2 + 3y^2 = 70y - 14y^2$$

$$25y^2 - 150y + 200 = 0$$
$$y^2 - 6y + 8 = 0$$
$$(y - 2)(y - 4) = 0$$
$$y = 2 \qquad\qquad y = 4$$
$$\downarrow \qquad\qquad \downarrow$$
$$x = 6 \qquad\qquad x = 2$$

ie $\qquad x = 2, y = 4 \qquad\qquad x = 6, y = 2$

Exercise 10

1. $p = aq - r$

$$aq = p + r$$
$$q = \frac{p + r}{a}$$

2. $k = 2mn$

$$m = \frac{k}{2n}$$

3. $d = \frac{5e}{t}$

$$td = 5e$$
$$t = \frac{5e}{d}$$

4. $F = \frac{srV}{a}$

$$Fa = srV$$
$$r = \frac{Fa}{sV}$$

5. $P = 5T^2 Q$

$$T^2 = \frac{P}{5Q}$$
$$T = \sqrt{\frac{P}{5Q}}$$

6. $y = 5t(x - 3)$

$$\frac{y}{5t} = x - 3$$

$$x = \frac{y}{5t} + 3$$

6. (Alternative)

$$y = 5t(x - 3)$$
$$y = 5tx - 15t$$
$$5tx = y + 15t$$
$$x = \frac{y + 15t}{5t}$$

(check the two answers are equivalent)

7. $y = 5t(x - 3)$

$$t = \frac{y}{5(x - 3)}$$

8. $B = \frac{3d^2 x}{C}$

$$BC = 3d^2 x$$
$$x = \frac{BC}{3d^2}$$

9. $B = \frac{3d^2 x}{C}$

$$BC = 3d^2 x$$
$$C = \frac{3d^2 x}{B}$$

10. $y = h - 0.2x$

$$0.2x = h - y$$
$$x = \frac{h - y}{0.2} = 5(h - y)$$

11. $R = x(3 - pt)$

$$R = 3x - xpt$$
$$xpt = 3x - R$$
$$t = \frac{3x - R}{xp}$$

12. $A = \frac{5(R - t)}{B}$

$$AB = 5(R - t)$$
$$B = \frac{5(R - t)}{A}$$

18.
$$A = \frac{5(R-t)}{B}$$

$AB = 5(R-t)$

$AB = 5R - 5t$

$5R = AB + 5t$

$R = \frac{AB + 5t}{5}$

19.
$$P = TV^2$$

$V^2 = \frac{P}{T}$

$V = \sqrt{\frac{P}{T}}$

14.
$$A = \frac{5(R-t)}{B}$$

$AB = 5R - 5t$

$5t = 5R - AB$

$t = \frac{5R - AB}{5}$

20.
$$a = 5(s^2 - r)$$

$\frac{a}{5} = s^2 - r$

$s^2 = \frac{a}{5} + r$

$s = \sqrt{\frac{a}{5} + r}$

15.
$$h = \frac{d}{3} + w$$

$h - w = \frac{d}{3}$

$d = 3(h - w)$

or (alternative)

$a = 5(s^2 - r)$

$a = 5s^2 - 5r$

$5s^2 = a + 5r$

$s^2 = \frac{a + 5r}{5}$

16.
$$w = y - cx$$

$cx = y - w$

$x = \frac{y - w}{c}$

$s = \sqrt{\frac{a + 5r}{5}}$

17.
$$A = 2\pi r^2 + 2\pi rh$$

$2\pi rh = A - 2\pi r^2$

$h = \frac{A - 2\pi r^2}{2\pi r}$

18.
$$V = abc$$

$b = \frac{V}{ac}$

21. $v = \dfrac{t+2}{t-2}$

$v(t-2) = t+2$

$vt - 2v = t+2$

$vt - t = 2 + 2v$

$t(v-1) = 2 + 2v$

$t = \dfrac{2+2v}{v-1}$

$ = \dfrac{2(1+v)}{v-1}$

22. $b = \dfrac{a-5}{3-a}$

$(3-a)b = a-5$

$3b - ab = a - 5$

$a + ab = 3b + 5$

$a(1+b) = 3b + 5$

$a = \dfrac{3b+5}{1+b}$

23. $h = \dfrac{t-2}{P-rt}$

$h(P-rt) = t-2$

$hP - rht = t-2$

$t + rht = hP + 2$

$t(1+rh) = hP+2$

$t = \dfrac{hP+2}{1+rh}$

Exercise 11

1. $\dfrac{g^3 h^2}{g^2 h^5} = \dfrac{g}{h^3} = gh^{-3}$

2. $\dfrac{3k^4 m^7}{9k m^2} = \dfrac{k^3 m^5}{3}$

3. $\dfrac{8n^5 p^2}{6n^7 p} = \dfrac{4p}{3n^2} = \dfrac{4}{3}pn^{-2}$

4. $\dfrac{q^{-2} r^3}{q\, r^4} = q^{-3} r^{-1} = \dfrac{1}{q^3 r}$

5. $(b^{-2})^3 = b^{-6}$

6. $(c^3)^{-2} = c^{-6}$

7. $(2d^3)^4 = 2^4 d^{12} = 16 d^{12}$

8. $(e^{-3})^{-2} = e^6$

9. $(ab^2 c)^3 = a^3 b^6 c^3$

10. $\left(\dfrac{2p^3}{4pq}\right)^2 = \left(\dfrac{p^2}{2q}\right)^2 = \dfrac{p^4}{4q^2} = \dfrac{p^4 q^{-2}}{4}$

11. $\left(\dfrac{3r^{-2}}{9r^2}\right)^{-1} = \left(\dfrac{r^{-4}}{3}\right)^{-1} = \dfrac{r^4}{3^{-1}} = 3r^4$

12. $(2a^3 b^{-3} c^2)^{-1} = 2^{-1} a^{-3} b^3 c^{-2} = \dfrac{b^3}{2a^3 c^2}$

13. $(4a^{-2} b^2 c^{-1})^2 = 16 a^{-4} b^4 c^{-2}$
$= \dfrac{16 b^4}{a^4 c^2}$

Exercise 12A

1) $\sqrt{60} = \sqrt{4}\sqrt{15} = 2\sqrt{15}$

2) $\sqrt{72} = \sqrt{36}\sqrt{2} = 6\sqrt{2}$

3) $\sqrt{18} = \sqrt{9}\sqrt{2} = 3\sqrt{2}$

4) $\sqrt{245} = \sqrt{49}\sqrt{5} = 7\sqrt{5}$

5) $\sqrt{48} = \sqrt{16}\sqrt{3} = 4\sqrt{3}$

6) $\sqrt[3]{48} = \sqrt[3]{8} \cdot \sqrt[3]{6} = 2\left(\sqrt[3]{6}\right)$

7) $\sqrt[3]{500} = \sqrt[3]{125} \cdot \sqrt[3]{4} = 5\left(\sqrt[3]{4}\right)$

9) $\sqrt[4]{162} = \sqrt[4]{81} \cdot \sqrt[4]{2} = 3\left(\sqrt[4]{2}\right)$

Exercise 12B

1) $\sqrt{3}\left(2+\sqrt{7}\right) = 2\sqrt{3} + \sqrt{21}$

2) $\sqrt{5}\left(1-\sqrt{2}\right) = \sqrt{5} - \sqrt{10}$

3) $\left(\sqrt{2}+1\right)\left(3+\sqrt{2}\right) = 3\sqrt{2} + 3 + 2 + \sqrt{2} = 5 + 4\sqrt{2}$

4) $\left(\sqrt{3}+2\right)\left(\sqrt{3}-1\right) = 3 + 2\sqrt{3} - \sqrt{3} - 2 = 1 + \sqrt{3}$

5) $\left(\sqrt{2}-2\right)\left(\sqrt{3}+5\right) = \sqrt{6} - 2\sqrt{3} + 5\sqrt{2} - 10$

6) $\left(\sqrt{2}+4\right)\left(\sqrt{3}-2\right) = \sqrt{6} + 4\sqrt{3} - 2\sqrt{2} - 8$

7) $\left(\sqrt{3}+\sqrt{2}\right)\left(\sqrt{3}-2\sqrt{2}\right) = 3 + \sqrt{6} - 2\sqrt{6} - 4 = -1 - \sqrt{6}$

9) $\left(3\sqrt{2}-4\right)\left(2\sqrt{2}-2\right) = 12 - 8\sqrt{2} - 6\sqrt{2} + 8 = 20 - 14\sqrt{2}$

9) $\left(\sqrt{5}+1\right)\left(2\sqrt{3}-3\right) = 2\sqrt{15} + 2\sqrt{3} - 3\sqrt{5} - 3$

10) $\left(\sqrt{2}+7\right)\left(\sqrt{2}-7\right) = 2 + 7\sqrt{2} - 7\sqrt{2} - 49 = -47$

Exercise 12C

1. $\dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{\sqrt{2}\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

2. $\dfrac{3}{\sqrt{5}} = \dfrac{3\sqrt{5}}{\sqrt{5}\sqrt{5}} = \dfrac{3\sqrt{5}}{5}$

3. $\dfrac{\sqrt{2}}{\sqrt{7}} = \dfrac{\sqrt{2}\sqrt{7}}{\sqrt{7}\sqrt{7}} = \dfrac{\sqrt{2}\sqrt{7}}{7} = \dfrac{\sqrt{14}}{7}$

Exercise 13

1. $x + 3 < 9$
   $x < 6$

2. $2x - 1 > 7$
   $2x > 8$
   $x > 4$

3. $4x \leq 32$
   $x \leq 8$

4. $3x + 2 \geq x - 6$
   $3x - x \geq -6 - 2$
   $2x \geq -8$
   $x \geq -4$

5. $2(x - 5) > 3x - 7$
   $2x - 10 > 3x - 7$
   $7 - 10 > 3x - 2x$
   $-3 > x$
   $x < -3$

6. $4 - 2x < x + 1$
   $4 - 1 < x + 2x$
   $3 < 3x$
   $x > 1$

7. $-2x + 3 > 9$
   $-2x > 6$
   $x < -3$

   (Division (or mult) by a negative number,
   change the direction of the inequality)

8. $7 - 8x \leq 1 - 2x$
   $2x - 8x \leq 1 - 7$
   $-6x \leq -6$
   $x \geq 1$

9. $2 + 3(x - 7) > 3 - 4(2x + 9)$
   $2 + 3x - 21 > 3 - 8x - 36$
   $11x > -14$
   $x > -\dfrac{14}{11}$

10. $5 - 3(2x - 1) \geq 4(1 - 3x) - 2$
    $5 - 6x + 3 \geq 4 - 12x - 2$
    $12x - 6x \geq 4 - 2 - 3 - 5$
    $6x \geq -6$
    $x \geq -1$